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Learning to Teach Productive Struggle: Elementary Preservice Teachers' Insights from Modeling in Mathematics

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ABSTRACT: *This study modeled the teaching practice of productive struggle for elementary preservice teachers (PSTs) within the context of teaching volume in mathematics. Drawing on social learning theory, it examined PSTs' conceptualizations of productive struggle and their interpretations of successful modeling indicators. Three university professors implemented productive struggle practices with fifty-nine elementary PSTs enrolled in mathematics methods courses, and data were collected through classroom observations and written surveys. The analysis was guided by Bandura's four processes of modeled learning: attention, retention, reproduction, and motivation. The findings revealed that questioning and encouragement were identified as the most successful indicators of productive struggle; however, discrepancies emerged between the indicators demonstrated in modeled practice and those reproduced by PSTs. In addition, PSTs anticipated significant challenges in implementing productive struggle, particularly in managing student frustration and fostering perseverance. These results provide important insights into how teacher education programs can better prepare PSTs to promote productive struggle effectively while equipping them with strategies to navigate the challenges of classroom implementation.*

Key words: *Elementary preservice teachers, mathematics education, modeling and observation, productive struggle, social learning theory.*

1. Introduction

The National Council of Teaching Mathematics outlines five process standards and eight mathematical practices that should be the core of all mathematics lessons. Within these processes and practices, NCTM recommends that students a) make sense of problems and persevere in solving them; b) reason abstractly and qualitatively; c) construct viable arguments and critique the reasoning of others; d) model with mathematics; e) attend to precision; f) look for and make use of structure and; g) look for and express regularity in repeated reasoning Although literature variously defines mathematical modeling (NCTM, 2014), we have adapted Hiebert and Grouws' (2007) definition of productive struggle as learning opportunities for students to make sense of mathematical ideas that are often challenging and not easy to figure out. Therefore, the primary constructs that allow students to succeed in a productive struggle include questioning, encouragement, ample time to struggle, and acknowledging their struggle (Mahovsky et al., in press). Productive struggle promotes students' communication about their mathematical thought and reasoning processes (Franke et al., 2015) and through teachers' purposefully planned scaffolding (Barlow et al., 2018).

What we refer to here as productive struggle is the ability of students to work through challenging problems that are not necessarily straightforward and require perseverance on these high-cognitive-demand

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tasks (NCTM, 2014; AMTE, 2017; Hiebert & Grouws, 2007; Rahman, 2022). This study examines how students interpret productive struggle as a means to model effective mathematics instruction. Through modeling, the professors provide short-term support to complete the task that learners might be unable to accomplish (Barlow et al., 2018). Guiding PSTs through their own struggle to see the balance between helping students and allowing them to immerse themselves in the productive struggle enables them to see the benefit for their future students (Rahman, 2022). Through this active learning, PSTs do mathematical tasks and think about their tasks (Solomon, 2007). Warshauer et al. (2021) contend that additional research is essential to examine how to help PSTs expand their mathematical interpretations of the struggles they notice among their students. Isolating when and how the PSTs experience productive struggle in their methods classes could determine how that changes their interpretation of a productive struggle. Through this literature review, the researchers focus on PSTs' prior experiences in learning mathematics, productive struggle, and real-world connections among mathematics through modeling, scaffolding, and collaboration.
The research questions for this study are:

1. How do preservice teachers (PSTs) interpret the modeling and effectiveness of a productive struggle?
2. How do they perceive the implementation of Productive struggles in their future

2. Literature Review

2.1. Theoretical Framework: Bandura's Social Learning Theory

Bandura's social learning theory provides a useful framework for understanding how PSTs learn productive struggle through observational learning and modeling. According to the theory, effective observational learning involves four key stages: attention, retention, reproduction, and motivation (Bandura, 1978). Applied to an educational setting—particularly in the teacher–learner relationship—this can be interpreted as follows: teachers model behaviors, learners attend to these behaviors (attention), internalize what they observe (retention), reproduce or replicate the modeled practices (reproduction), and require motivation to enact and sustain what they have learned (motivation) (Rumjaun & Narod, 2025). The relevance of these stages to the present study is illustrated in Figure 1.

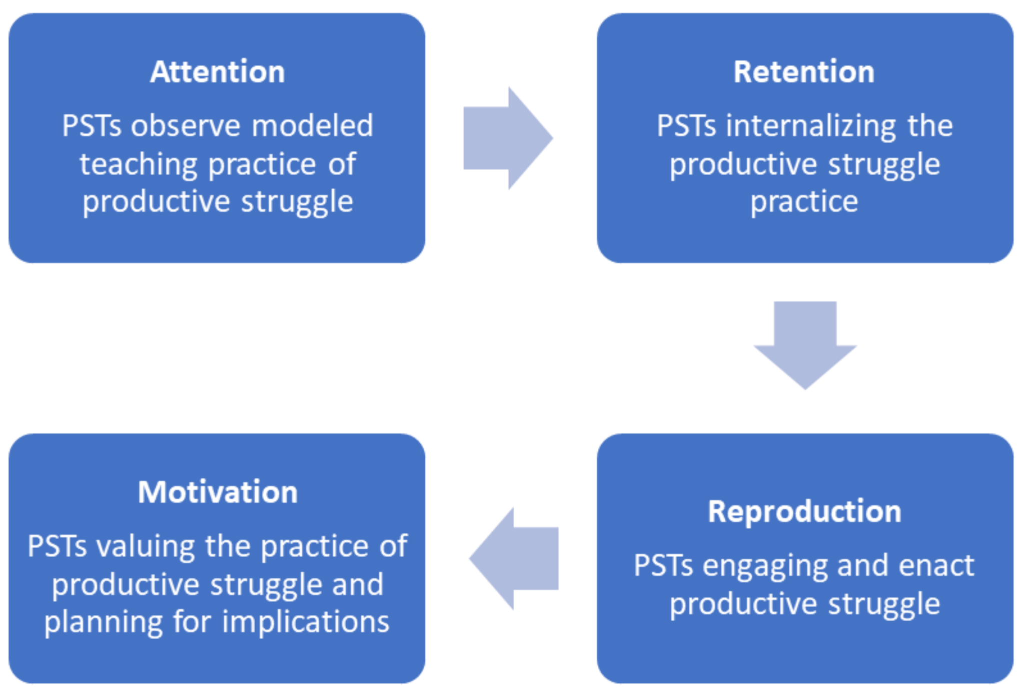

Figure 1. Learning process of modeling productive struggle.

2.2. Modeling Among Preservice Teachers

Schutz & Rainey (2020) define modeling as "both demonstrating and thinking aloud to make a process visible so students can learn to engage in the same process (p.444). However, teachers' personal experiences influence their beliefs about mathematics teaching and learning and their perceptions of a student's

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mathematical disposition (Wilhelm, 2014). Research studies about mathematical modeling in teacher education have centered predominantly on secondary teachers versus elementary teachers (Tidwell, 2021). Therefore, this study of elementary PSTs utilizing effective intervention strategies, mathematical knowledge for teaching, and their willingness to implement mathematical modeling in their classrooms is imperative (Tidwell, 2021). Consequently, requiring teachers to take more math content or methods courses may improve their mathematical and pedagogical knowledge, but it will not automatically influence PST's beliefs and awareness of teaching conceptually (Lawrence et al., 2014; Munter & Correnti, 2017). According to Tidwell et al. (2023), interventions addressing a PST's pedagogical content knowledge are essential for mathematics teachers to attain the necessary mathematical knowledge for teaching modeling. For this reason, it is essential to work with PSTs to develop a robust and refined vision of high-quality mathematics instruction, choosing tasks that allow their students the most significant opportunities for conceptual understanding of mathematical tasks (Wilhelm, 2014). Solomon (2007) asserts that relating mathematics to context is a fundamental component of the mathematical experience for PSTs. As a result, when interventions are done with elementary PSTs, their ability to perform mathematical modeling tasks improves (Tidwell et al., 2023). At the same time, these introduced interventions must interact with and, in some cases, replace old practices, which are usually deeply rooted ways PSTs have interacted with students (Munter & Correnti, 2017). Munter and Correnti (2017) contend that PSTs, in the transition of learning new practices, are not only transitioning to new forms of practice but trying to suppress practices they have either experienced or observed. For this reason, teachers' knowledge and perceptions are interconnected with their ability to enact cognitively demanding tasks (Wilhelm, 2014). Thus, Wilhelm (2014) argues that it is essential to concurrently develop the many aspects of a PST's knowledge, conceptions, and practice. Practical support for PSTs should center on problems of practice and encourage them to discuss and develop their mathematical knowledge for teaching and perceptions of teaching and learning mathematics (Wilhelm, 2014).

2.3. Productive Struggle among Preservice Teachers

Through the modeling process of productive struggle, PSTs must understand the problem situation, research the problem, reconsider the problem, formulate a model to solve the problem, interpret the results, validate the results, revise, refine, and, therefore, notice that this repetitive process is expected in modeling mathematics (Tidwell et al., 2023). According to Polly (2017), great potential exists for PSTs to rebuild their mathematical understanding by exploring cognitively demanding mathematical tasks through productive struggle. As teachers work with struggling students in the classroom, they are more likely to maintain the cognitive demands of high-level tasks. However, their views on supporting these struggling students are unrelated to choosing a high-level task (Wilhelm, 2014). With this understanding, a PST that models and then provides direct instruction at the beginning of the lesson hinders a student's ability to engage in a productive struggle in solving high-level tasks (Munter & Correnti, 2017). As PSTs transition from their conceptions of teaching and learning mathematics to an inquiry-oriented perspective but are not quite sophisticated in their understanding of productive struggle, they often agonize about enacting their desired instructional practices (Wilhelm, 2014). Solomon's (2007) findings showed that PSTs experienced having their boundaries pushed as they worked independently from the professor to struggle productively. For PSTs to push through this, they needed to be open-minded while actively seeking reassurance, with support from their peers, that they were correct in their thinking (Solomon, 2007). Through this productive struggle, mathematical modeling is enmeshed with taking a real-world situation and creating a mathematical task from it (Spooner, 2022).

2.4. Scaffolding Instruction for Productive Struggle

Determining the task for instruction is only the first step; knowing how to scaffold the lesson is paramount in the modeling process. To provide access to productive struggle, teachers must introduce scaffolding before students engage with the task (Barlow et al., 2018). Scaffolding is defined as temporary support that enables students to access productive struggle that might otherwise be unattainable. As Barlow et al. (2018) note, "Purposefully planned scaffolding can determine whether students struggle productively or just simply struggle" (p. 206).

In parallel, modeling involves active learning through assisted discovery, creation, and collaboration (Solomon, 2007). Polly (2017) further argues that instructional plans emphasizing student engagement with real-world problem solving prior to teacher modeling lead to gains in both problem-solving ability and student

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engagement within the mathematics lesson. Collectively, these studies underscore the importance of scaffolding and modeling in ensuring that students' initial engagement with tasks leads to meaningful productive struggle rather than unproductive frustration.

3. Methodology

3.1. Participant

This study included 59 elementary PSTs in three sections of a semester-long mathematics methods course, emphasizing mathematics instruction through group collaboration, a real-world task, conceptual understanding, and hands-on experiences. The researchers were the university instructors of the three sections of the course, and the university was located in the Midwest region of the United States. This study examined PSTs' interpretation of productive struggle while engaging modeled productive struggle activity. The researchers achieved this goal by employing modeling and scaffolding teaching strategies that centered around the concept of volume.

3.2. Productive Struggle Activity and Modeling

This activity was designed to provide preservice teachers (PSTs) with opportunities to engage in productive struggle while constructing a conceptual understanding of volume. Related research shows that PSTs often hold the misconception that folding the same-sized piece of paper will always result in the same volume (Mahovsky et al., 2025). To address this misconception, we designed a productive struggle experience aimed at enhancing their conceptual understanding of volume. A team of three researchers collaboratively developed the activity guidelines, which specified the mathematical focus (volume), instructional approaches, supporting materials, guiding and reflection questions, and observation protocols. Each professor then implemented the activity within their respective mathematics methods courses, adhering to the shared framework. This framework allowed a consistent yet contextually rich examination of PSTs' engagement.

The prompt was: *If you have two sheets of paper of the same size and want to fold one to hold materials such as beans, which 3D fold would hold more?* PSTs then used hands-on materials—including letter-sized paper to create 3D shapes (e.g., long or short cylinders, long or short rectangular prisms), pinto beans, measuring cups, rulers, and tape—to investigate which fold could hold more. Through this activity, they explored multiple strategies to determine volume while researchers modeled instructional approaches that encouraged productive struggle.

The goals of this activity were twofold. First, PSTs were able to meaningfully experience productive struggle as learners, consistent with NCTM's (2014) description of acknowledging and utilizing struggle as an opportunity to learn mathematics (p. 49). Second, PSTs observed how productive struggle can be modeled for future teaching. This modeled practice included, but was not limited to, providing opportunities to struggle with the problem, observing participants' behaviors and struggles, posing and responding to questions, and clarifying processes.

3.3. Data Collection

Under the structured guidance of the researchers, PSTs collectively participated in and reflected on the task, while the researchers documented classroom conversations, observational notes, and collected reflection responses. After completing the task, preservice teachers (PSTs) participated in a survey consisting of open-ended questions about their conceptualization and experiences of productive struggle. Guided by Bandura's social learning theory, the survey was structured around four main themes—attention, retention, reproduction, and motivation—as illustrated in Figure 1. These themes represent the sequential stages of observational learning and provided the framework for examining how PSTs interpreted and internalized productive struggle. They were particularly important for understanding how PSTs internalized the practice and how they might reproduce it in their future classrooms. Table 1 presents the reflection questions along with the rationale for each.

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Table 1. Reflection Questions and Rationales.

Theme	Written Reflection Questions	Rationales
1. Attention	1. What did a teacher do during this productive struggle instruction?	To capture what is being modeled and observed as indicators of teachers' productive struggle practices.
2. Retention	2.1) What is productive struggle? 2.2) What are productive struggles like in a mathematics classroom?	To represent how PSTs conceptualize and internalize productive struggle.
3. Reproduction	3. What did you do as a student during PS instruction?	PSTs' reflections on their own behaviors serve as indicators of students' productive struggle experiences.
4. Motivation	4.1) What are the benefits of a productive struggle? 4.2) What challenges do you foresee bringing a productive struggle into your future classroom?	To represent PSTs' productive struggle practice implications for their future teaching.

3.4. Analysis

The data were coded through open, process, focused, axial, and theoretical coding (Saldaña, 2015). The data analysis encompassed a two-step coding cycle: within and across each case (Yin, 2009). First, the three researchers compiled survey responses from all sections into a spreadsheet to enter data. Beginning with their classroom data, they reviewed the data multiple times to find emerging themes. After the first round of coding, the researchers convened to discuss emerging themes across the sections and resolve any ambiguities in responses. The iterative process was repeated to achieve a consensus of 90%, aiming to enhance credibility (Merriam, 1998) and attain a higher inter-coder agreement (Saldaña, 2015).

Regarding the data analysis of modeled productive struggle, it was important to demonstrate the authenticity of modeled practice. To assess this, PSTs were asked what the researchers did during the productive struggle activity (Q1 above), and their responses were reviewed and aligned with teachers' indicators of successful modeling of productive struggle, revised from Warshauer (2015). As indicators represent observable behaviors and signs of engagement in productive struggle, the researchers analyzed participants' responses using Warshauer's (2015) rubric and calculated frequencies to identify patterns and prevalence.

For example, when a PST described teacher behavior in Q1 as "have students make a prediction, explain the reasoning," the response was coded under the *questioning* indicator. Similarly, when a response stated, "asked leading questions when we were stuck," it was coded as *encouraging*. It is noteworthy that a single response could be assigned multiple indicators. For instance, a PST's statement, "teacher provided materials and had us make a prediction / came over to ask essential questions / made us justify our thinking," was coded as both *giving time to struggle and grapple with content* and *questioning*. Similarly, to assess how PSTs experienced the productive struggle process (*Retention Q3*) as learners, we repeated the same procedure, coding their responses using student indicators of productive struggle. The full rubric is presented in Table 2.

However, the analysis process differed for Q2 and Q4, as these questions reflected participants' thinking rather than observed behaviors. Given the wide variation in PSTs' statements, identifying consistent emerging themes proved challenging. Instead, the analysis focused on recurring keywords. Frequently mentioned terms included *perseverance, critical thinking, questioning, learning from the process, conceptual understanding, teacher support,* and *collaborative learning.* The researchers then concentrated on the top four to five keywords that appeared most often, using these to present the results. A similar procedure was applied to analyze responses regarding the potential implications of productive struggle (Q4).

This iterative coding process was repeated until a 90% consensus was reached, enhancing credibility (Merriam, 1998) and ensuring higher inter-coder agreement (Saldaña, 2015).

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Table 2. Indicators of Successful Modeling of Productive Struggle.

Themes	Teacher Indicators of a Productive Struggle	Student Indicators of a Productive Struggle
Questioning	Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem.	Students ask questions to identify the source of their struggle, write down their ideas, clarify ideas with others, and consider alternative strategies or representations to address their struggle.
Encouraging	Teachers encourage students to reflect on their work and support students who struggle in their effort versus the process of only getting correct answers.	Students use their effort to solve problems and try to make sense of their work, not only satisfied with a correct answer or that they perceive themselves as smart or not.
Give Time to struggle and grapple with content.	Teachers give time and support for students to manage their struggle through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students.	Students use their time to develop and follow through on their strategies, evaluate their progress, and understand what they can do and what remains to be done.
Teachers acknowledge Learning	Teachers acknowledge that struggle is an integral part of learning and doing mathematics.	Students persist in their work to make sense of and solve problems without giving up or getting discouraged easily.

Note: *This table was adopted and revised from Warshauer (2015).

4. Results

In this study, PSTs were given opportunities to observe professors' modeling of productive struggle, to experience it as learners, and to reflect on it in order to consider future implications. Accordingly, the results are organized around the four dimensions of attention, retention, reproduction and motivation.

4.1. Attention – Teacher Indicator of Productive Struggle.

The frequency analysis of Q1 revealed that the most frequently identified teacher indicator was *questioning*, which appeared in 92% of PSTs' statements. PSTs described questioning as a strategy that helps learners locate the source of difficulty and focus on problem-solving strategies. The second most frequent indicator was *encouragement* (70%), where teachers were perceived as motivating students to reflect on their work and persist in problem-solving rather than seeking immediate answers. In addition, 53% of PSTs noted the indicator *giving students time to struggle and grapple with content.* However, no responses indicated that teachers explicitly *acknowledged struggle as an integral part of learning and doing mathematics.*

4.3. Retention: Internalizing Productive Struggle

To examine PSTs' conceptualization of productive struggle, participants were asked to explain (a) what productive struggle is and (b) what it looks like in a mathematics classroom.

For the general concept of productive struggle, five themes emerged: *learning from struggles and challenges*, *perseverance*, *teacher questioning and guidance*, *justifying answers*, and *critical/deep thinking.* These responses suggest that PSTs' understanding aligns closely with key elements of productive struggle discussed in prior research.

When asked to describe what productive struggle looks like in a mathematics classroom, six major themes emerged. These were closely related to the general themes identified above but also included distinctive mathematics classroom-specific features such as *multiple solutions/strategies* and *group work/collaboration.* Many PSTs highlighted these as essential elements of mathematical productive struggle. Below are selected responses:

- "In a classroom, a math productive struggle helps students gain a deep, conceptual understanding of different concepts by working through *different solution methods*" (PST 7).

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- "Math productive struggle involves presenting an open-ended problem and letting students try *various solutions* to solve it" (PST 39).
- "Math productive struggle looks like students working together in *groups*, and discussing *ideas and strategies*"(PST 42).
- "It looks like students working in *groups* and bouncing ideas off each other". (J12) "Allowing students to *work together* and struggle"(PST 46).
- "This is when a student is struggling with math concepts during a lesson. The teacher proposes an issue for the class to *solve as a group*"(PST 15).
- "Students are explaining how they solved something, *working together*, and move through problems *together*" (PST 21).
- A math productive struggle should look like collaboration, and others such as wait time, questioning, encouragement, patience, and perseverance (PST 35, 36 & 37)

4.4. Reproduction - PSTs Practicing Productive Struggle as Learners

This section reports how PSTs reflected on their experiences of productive struggle and compares these reflections with teacher modeling to identify differences. The analysis revealed that PSTs most frequently experienced productive struggle through *teachers give time to struggle and grapple with content* (87 %), the *questioning* indicator (65 %), followed by *encouragement* (38%), and *acknowledging learning* (36%).

Table 3 compares the frequency of statements denoting successful indicators of productive struggle across two groups: the three professors who modeled productive struggle as teachers and the preservice teachers (PSTs) who engaged with the practice as learners. While both groups emphasized similar indicator themes, the distribution of frequencies differed.

Table 3. Percentage of frequency: Indicators of successful modeling of productive struggle.

Indicators of Productive Struggle	**Retention /Observation What PSTs noticed in modeled PS practice**	**Reproduction /Practice What PSTs experienced as learners about PS practice**	**Average Observation Frequency**
Questioning	92%	65%	78.5%
Encouraging	70%	38%	54 %
Teachers give time to struggle and grapple with content	53%	87%	70%
Teacher acknowledges learning	0%	36%	18%

As shown in Table 3, the frequency order of indicators differed between teachers and students. While teachers most frequently identified *questioning* (92%) as the central indicator of productive struggle, PSTs also recognized questioning as important (65%), though to a lesser extent. From the learners' perspective, however, the most salient indicator was *grappling with content* (87%), which aligns with the teachers' theme of *giving time to struggle* (53%).

The second most frequent indicator among teachers was *encouragement* (70%), yet only 38% of PSTs reported experiencing encouragement as part of their productive struggle. Similarly, *acknowledging struggle as a learning opportunity* was the least represented indicator overall. While 36% of PSTs noted experiencing this indicator, none identified it explicitly in professors' modeled teaching (0%). The combined analysis of both groups revealed that the most frequently identified indicators of productive struggle, in descending order, were *questioning*, *grappling with content*, *encouragement*, and *teacher acknowledgement.*

4.5. Motivation- PSTs Valuing Productive Struggle and Planning for Implications

In the last question 4.1, PSTs addressed the benefits and challenges of incorporating productive struggle into their future classrooms. The analysis notably indicated that the most significant benefit of productive struggle was its role in promoting critical thinking. They expressed that this productive struggle supported their thinking about what volume is and why different types of folds, made from the same-sized paper, could

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hold more or less. It also highlighted the factors that contribute to volume and thereby their conceptual understanding of volume. This keyword of critical thinking exhibited a higher frequency than other emerging keywords, such as justification, perseverance, problem-solving skills, and communication teamwork. Subsequently, in question 4.2, the researchers sought insights into the challenges of implementing these teaching techniques in the classroom and three predominant themes surfaced across 59 PSTs: ' students getting frustrated/giving up,' 'lack of understanding/guiding students' thinking,' and 'letting students struggle/teaching patience.' Additionally, a handful of PSTs mentioned concerns related to classroom management, unproductive struggle, differentiated students' level, overwhelming, and so on. The PSTs identified students' frustration as the most prevalent challenge in carrying out productive struggle, and this theme significantly outweighed the frequency of other challenges mentioned above. The following statements encapsulate the sentiments expressed by PSTs in addressing these challenges:

- "Students getting *frustrated* and upset,"
- "Students crying or refusing to continue work."
- "Students easily *giving up*"
- "I can foresee students *giving up* immediately."
- "Students becoming *frustrated* by not receiving the answer."
- "When students get *frustrated*, they can either *give up* or lash out."
- "I would be worried about my students getting *frustrated* very quickly."

5. Discussion

5.1. Discrepancy between What is Being Modeled and What PSTs Experienced

As summarized in Table 3, the findings highlight discrepancies between indicators of productive struggle across two groups: PSTs' reflections on teachers' modeled productive struggle and their own experiences as learners. The frequency distribution illustrates that teachers effectively modeled productive struggle through *questioning, encouragement, providing time to grapple with content,* and *acknowledging learning.* For teachers, questioning emerged as the dominant indicator, while learners prioritized their lived experience of grappling with content. These complementary perspectives—pedagogical focus versus learner experience—point to both strengths and gaps in how productive struggle is modeled and perceived in elementary teacher education.

Learners' emphasis on grappling with content reflects their focus on pursuing strategies, monitoring progress, and recognizing what they could and could not yet accomplish. In contrast, teachers emphasized questioning as their primary pedagogical move. This discrepancy is understandable: teachers naturally attend to instructional actions, whereas learners focus on their personal experiences of struggle. Still, it raises important questions about how modeling can be designed so that PSTs are able not only to observe but also to transfer and reproduce these practices in their own teaching. Effective training happens when educators have experienced productive struggle in the role of a learner and reflect upon what the teacher and student did during the experience. The experience of productive struggle should be integrated into teacher training programs to develop a shared understanding among educators.

Meanwhile, both groups placed relatively little emphasis on acknowledging struggle as a valuable part of learning. Notably, although 36% of PSTs reported experiencing this indicator, none observed it explicitly in the teachers' modeled practice (0%). This finding underscores the need for teacher educators to encourage PSTs to adopt a mindset that values the benefits of productive struggle and to normalize the idea that struggle is necessary when engaging in challenging tasks. Explicit verbal assurance, reflective classroom activities, and intentional modeling can all help reinforce this perspective.

5.2. Important Features of Productive Struggles in Mathematics Classroom

In this study, PSTs identified common themes related to the general concept of productive struggle. However, when applied specifically to the mathematics classroom, additional themes emerged—most notably, the importance of *multiple solution strategies* and *group work/collaboration.*

The findings first underscore the value of encouraging multiple solution strategies as a way to engage with productive struggle. When mathematics instruction is narrowly focused on obtaining the correct answer, it may inadvertently foster math anxiety and a fear of making mistakes. In contrast, when students are

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encouraged to explore a range of reasonable solutions, they are more likely to engage actively, persist through challenges, and develop conceptual understanding.

A second key theme centered on group work and collaboration. Prior research emphasizes that collaboration and communication are vital to mathematics learning (Nilimaa, 2023). Collaborative problem solving encourages students to remain open-minded and to consider diverse perspectives (Solomon, 2007). During instruction, this requires students to listen attentively, reflect on others' ideas, and integrate multiple approaches, thereby enabling them to make progress in their learning (Nilimaa, 2023; Solomon, 2007). Through such collaboration, students experience flexibility in problem solving and openness to new strategies, which involves being actively engaged in the task, connecting mathematics to real contexts, working with peers, and testing alternative strategies (Solomon, 2007).

Taken together, these findings highlight that future educators should be trained not only to recognize productive struggle but also to facilitate mathematical productive struggles that emphasize collaboration and the exploration of multiple solution strategies.

5.3. Navigating PSTS' Challenges of Future Implications

The findings of this study highlight that PSTs valued the importance of productive struggle across multiple dimensions of learning, yet at the same time anticipated difficulties in implementing it. Their primary concerns centered on student frustration and the need to foster patience in the face of struggle. Rahman's (2022) study further underscores this challenge, demonstrating that even experienced teachers find it difficult to balance providing support with allowing students to engage meaningfully in productive struggle.

Recognizing the challenges that PSTs foresee is therefore essential for preparing them in advance. Anticipating obstacles encourages PSTs to consider strategies for addressing student frustration, such as designing scaffolds, posing guiding questions, and employing effective management approaches. In doing so, PSTs can build confidence and resilience, reframing challenges as opportunities for growth rather than sources of discouragement. Their own experiences of productive struggle as learners can further support this shift in mindset, enabling them to view struggle positively for both themselves and their future students.

6. Conclusion

Drawing on Bandura's social learning theory (1978), this study explored (1) how preservice teachers (PSTs) interpret the modeling and effectiveness of productive struggle and (2) how they perceive its implementation in their future classrooms. The findings indicate that PSTs internalized productive struggle conceptually and engaged with it meaningfully as learners. At the same time, discrepancies emerged between what was modeled by teachers and what PSTs experienced personally. In particular, acknowledging struggle as a valuable learning process was weak for both groups.

Overall, the results suggest that while modeling is a necessary component of teacher preparation, it is not sufficient on its own. PSTs emphasized the importance of experiencing productive struggle themselves, which enabled them to reflect more deeply on both teacher and student roles in the process. This highlights the need for teacher educators to provide structured opportunities for PSTs to grapple with challenging tasks, reflect on their learning, and consider the implications for elementary mathematics instruction.

In addition, the findings point to two key features of mathematical productive struggle that require greater emphasis in teacher education: encouraging multiple solution strategies and facilitating group work and collaboration. Both themes are essential for helping students view struggle as a natural and productive part of mathematics learning rather than a source of anxiety or discouragement.

Finally, this study underscores the importance of preparing PSTs not only to recognize productive struggle but also to anticipate the challenges of implementing it—such as managing student frustration and sustaining perseverance. Teacher preparation programs and curriculum developers should align training with these expectations, and extend support into the induction years, when novice teachers are most likely to encounter the complexities of productive struggle in the classroom.

7. Limitations and Study Forward

This study is qualitative in nature with a small sample size and focused on a single activity related to volume, which limits the generalizability of the findings. Future research should expand to include a broader range of mathematical tasks and larger, more diverse populations of preservice teachers. In addition, further

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studies are needed to investigate how modeling that explicitly emphasizes encouragement and normalization of productive struggle—combined with clear guidelines for managing students' frustration—can support the effective implementation of productive struggle in mathematics classrooms.

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Authors' Contribution:

All three authors jointly designed the study, collected the data, and collaborated in the analysis. Authorship order was determined based on the relative amount of writing contribution. The first author contributed the largest portion of the writing and led the revision process.

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