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Stock Buybacks: An Option-Based Mathematical Modeling Approach to Financial Decision-Making

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ABSTRACT: *The share repurchase decision is strategic for large corporations, yet comparative studies integrating mathematical modeling, financial options, and portfolio optimization remain limited. This study addresses this gap by proposing a quantitative analysis of share repurchase decisions using multiple approaches. The research compares the profitability of a Brazilian bank's share repurchase program with the allocation of the same capital into an efficient portfolio, constructed based on DEA BCC models, Markowitz, Monte Carlo simulation, and financial option theory. Binomial modeling and the Black-Scholes model are applied to value repurchase as a financial option, incorporating risk, flexibility, and volatility. Data were processed in Python using historical time series from B3. The results show that, in the analyzed scenario, the optimized portfolio financially outperforms repurchase, providing a more efficient alternative. The study offers practical and technical evidence to support complex corporate decisions in highly uncertain environments.*

Key Words: DEA, financial option, mathematical modeling, portfolio optimization, share repurchase.

1. Introduction

Corporate capital allocation decisions are crucial for firms' performance and valuation in financial markets. Among these decisions, share repurchase stands out as a strategic alternative that can influence both stock prices and the company's capital structure (WU; CHAN; CHUNG, 2022). Repurchase programs are commonly used by firms with excess cash that wish to signal confidence to the market, reduce the number of outstanding shares, and improve financial indicators such as earnings per share (JAMADAR et al., 2024). Moreover, buybacks may directly affect investors' perception of risk and return (APERGIS et al., 2021). Proper evaluation of these strategies is essential to ensure alignment with the organization's long-term objectives. Studies such as Cook and Zhang (2022) emphasize that repurchase should be assessed as a financial option, taking into account uncertainty and volatility. In this context, it becomes relevant to investigate whether share repurchase indeed represents the best allocation of resources.

As an alternative to repurchase, allocating excess capital into an efficient stock portfolio can maximize risk-adjusted returns for the company. Markowitz's mean-variance model (1952) remains one of the main references for building optimized portfolios, with extensive application in finance studies (MARTÍNEZ-SÁNCHEZ; BERRONES-SANTOS; ALMAGUER MARTÍNEZ, 2023). The choice of assets that compose the portfolio depends on analyzing variables such as expected return, volatility, and

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correlations among assets—elements that are fundamental in constructing efficient frontiers (CHIBANE; JOUBREL, 2024). This type of decision involves both quantitative and strategic aspects and requires simulations to account for future scenarios. Comparing alternatives—such as diversification versus repurchase—has been the subject of studies analyzing net present value, risk, flexibility, and value creation (ONALI; MASCIA, 2022). Therefore, it is also necessary to consider additional tools that can assist decision-making under uncertainty.

To support investment decisions in uncertain environments, methods such as Monte Carlo simulation and financial option analysis have been widely applied (ZAKRZEWSKA-BIELAWSKA; LIS; UJWARY-GIL, 2022). Monte Carlo simulation allows modeling asset behavior under multiple scenarios, accounting for distributions of returns and volatility, which is especially relevant in financial market decisions (BLASQUES; LUCAS; VAN VLODROP, 2021). DEA, in turn, is a non-parametric frontier technique that evaluates the relative efficiency of decision-making units with multiple inputs and outputs, and in this study, it is used to identify the most efficient stocks in the market (EBRAHIMI; HAJIZADEH, 2021).

Despite the relevance of these models, most studies still analyze repurchase decisions in isolation, without comparing them to other investment alternatives that may provide better risk-adjusted returns. Furthermore, many corporate decisions are made based on short-term financial indicators, disregarding managerial flexibility and resource allocation efficiency (ZHU; ZHOU; LIU, 2023). The integration of models can enable a more robust and strategic analysis, offering stronger support for corporate decision-making. Research such as that by Mohammadi Jarchelou, Fathi Vajargah, and Azhdari (2023) highlights the need for integrating quantitative techniques in investment evaluation. The absence of studies combining portfolio models, financial options, and DEA demonstrates a methodological gap in the analysis of share repurchase decisions. Given this context, the following research question arises: in a scenario of excess cash, would it be more advantageous for Banco Itaú to repurchase shares or to invest in an efficient asset portfolio? Between May 2023 and November 2024, the company repurchased 10 million of its own shares.

Studying this decision is relevant from both practical and academic perspectives, as it involves the use of robust methodologies that can be replicated by firms with similar profiles. The use of real data, combined with advanced quantitative modeling, provides a reliable foundation for simulating scenarios and supporting strategic decisions. From an academic perspective, the integration of DEA, Monte Carlo simulation, financial option models, and portfolio optimization represents a methodological innovation that remains underexplored in the literature. Previous studies have treated these methods separately, but their combination can yield more comprehensive and realistic results. The present study aims to compare the economic viability of share repurchase with the allocation of the same capital into an efficient portfolio, using an integrated financial evaluation and decision-support model.

The structure of this study is organized as follows: the next section presents the theoretical framework. This is followed by the methodological procedures, detailing data sources, asset selection criteria, portfolio construction steps, and the parameters of option and simulation models. The subsequent section presents the results obtained from the application of DEA, Markowitz, Black-Scholes, and binomial models. The discussion section provides a comparative analysis of the results in light of theory and corporate practice. Finally, the concluding section summarizes the main findings, managerial contributions, study limitations, and suggestions for future research.

2. Theoretical Framework

2.1. Financial Market and Share Repurchase Strategies

The dynamics of the modern financial market are characterized by increasingly sophisticated corporate decisions that seek to balance shareholder return with strategic sustainability (CAVALCANTE et al., 2016). At the core of these decisions lies the efficient use of equity capital, particularly in large firms with strong cash generation (YASAR, 2021). A highly liquid and competitive environment requires companies to make swift decisions on how to allocate their financial resources (LI; OKUR, 2023). Among these strategies, share repurchase stands out for its effects on both market value and financial indicators of the firm (WU; CHAN; CHUNG, 2022). Herskovits, Muhle-Karbe, and Tse (2025)

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reinforce that this practice is common in mature companies and is often used as an alternative to dividend distribution.

Share repurchase can serve multiple purposes, such as signaling undervaluation, adjusting capital structure, or providing executive incentives (GOPAL et al., 2024). Studies such as Hansen and Andersen (2023) show that repurchases are frequently associated with superior long-term performance. The literature also suggests that the practice positively affects stock prices by reducing the number of shares outstanding (CHEN; LIU, 2023). However, this strategy requires careful analysis, as it may compromise the company's future liquidity (BARGERSTOCK; ABBASI, 2022). In emerging markets such as Brazil, this practice has been consolidating gradually, as indicated by Saxena and Sahoo (2023).

In Brazil, the stock market has shown growth and maturity in recent years, driven by the increasing number of investors and the consolidation of large financial institutions (FONSECA; VAN DOORNIK, 2022). This scenario has favored the adoption of strategies such as share repurchase, particularly by banks and companies with strong cash generation (LAVINAS; ARAUJO; GENTIL, 2022). However, analyzing repurchase in the Brazilian context requires considering variables such as cost of capital, market volatility, and regulatory frameworks (ZHANG; TONG; JIN, 2023). This reinforces the importance of robust models to support managerial decision-making.

The literature suggests that, under uncertainty, financial decisions can be treated as financial options (HE; LIN, 2021). This means modeling repurchase as a financial decision that the company may or may not exercise depending on future market conditions. This approach is supported by authors such as Khan et al. (2023), who view real options as an effective tool for corporate decisions under risk. Repurchase can be modeled as an American call option, since the firm may exercise it at any moment during the validity period (SONG et al., 2022). Such modeling allows for the incorporation of aspects such as volatility, present value of cash flows, and managerial flexibility (HUSSAIN et al., 2023).

For profitability to be ensured, it is necessary to evaluate the opportunity cost of financial operations (CORTES, 2021). Among the alternatives is the allocation of resources in stock portfolios, as proposed by Markowitz in 1952 (JOBSON; KORKIE, 1980). This approach makes it possible to assess portfolio recomposition based on risk and return, offering a solid comparative basis (KONNO, 1990).

In summary, the literature shows that share repurchase is a multifaceted strategy that must be analyzed alongside other capital allocation alternatives. For this purpose, the use of quantitative methods and robust financial models is recommended (OSTERRIEDER, 2023). This study is positioned within this context, proposing a comparative analysis between share repurchase and investment in an efficient portfolio. The central research question is: how should firms analyze the trade-off between allocating excess cash to share repurchases versus to an optimized market portfolio?

2.2. Quantitative Tools in Corporate Investment Evaluation

The use of quantitative tools is a well-established practice in investment evaluation, particularly in contexts of uncertainty and multiple scenarios (HAJER, 2024). Among these tools are portfolio optimization models, Monte Carlo simulation, and option pricing models. These methods allow for the incorporation of variability, risk, and return into analyses, offering more robust support for decision-making (MUROI; SUDA, 2022; JIANG; HAO; SUN, 2025). In the corporate context, quantitative analysis is fundamental for choosing between alternative capital allocation strategies (UYAR et al., 2023). Studies show that model-driven decisions increase assertiveness and mitigate risk (EMROUZNEJAD; ABBASI; SICAKYÜZ, 2023).

Modern portfolio theory, proposed by Markowitz, introduced the concept of optimal asset allocation based on the balance between risk and return. This model has been widely disseminated and refined by several authors, such as Li et al. (2022), and applied in various investment analyses. The covariance matrix, expected returns, and asset volatility are central elements in the formulation of the efficient portfolio (MYNBAYEVA; LAMB; ZHAO, 2022). The use of the optimal portfolio provides a measure of opportunity cost for resource allocation studies (SAGHEZCHI; KASHANI; GHODRATIZADEH, 2024).

For share repurchase, modeling as a call option is performed with American characteristics, allowing exercise at any moment, unlike European options (YAN et al., 2022). The main models employed are the binomial model (Cox; Ross; Rubinstein, 1979) and the Black-Scholes model (RATIBENYAKOOL;

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NEAMMANEE, 2024). These models incorporate variables such as time, volatility, interest rate, and strike price (LIU; ZHU, 2024).

The analysis also involved the use of the DEA technique, which allows evaluating the relative efficiency among Decision-Making Units (DMUs) (Charnes; Cooper; Rhodes, 1978). This linear programming method is useful for selecting investments that not only maximize returns but also perform well in other indicators (BEN LAHOUEL et al., 2024). This combination is particularly relevant when data are abundant and asset selection can strongly influence portfolio outcomes. The integration between DEA and Markowitz, as proposed by Ebrahimi and Hajizadeh (2021), provides a more robust framework for portfolio construction.

3. Methodological Procedures

This study was developed through simulations and mathematical and computational modeling applied to historical stock data traded on B3 (the Brazilian Stock Exchange), with the objective of evaluating the feasibility of Banco Itaú's share repurchase program. The study analyzes two scenarios: (i) the execution of the repurchase, modeled as an American real call option, and (ii) the alternative of allocating resources into an efficient stock portfolio, constructed based on Markowitz's mean-variance model. Initially, stocks were subjected to an efficiency analysis through the DEA (Data Envelopment Analysis) technique to select the most suitable assets for portfolio formation. The choice of Banco Itaú is justified by its recurring history of buybacks and its strategic importance in the financial sector (PACHECO; BRANCO, 2025). References such as Demirag, Kungwal, and Bakkar (2022) reinforce this practice among firms with strong cash generation. The analysis period spans from May 2023 to November 2024, during which Banco Itaú conducted share repurchases.

The data used were obtained from reliable secondary sources, such as Yahoo Finance and B3, comprising daily stock prices of Itaú and other companies listed on the exchange. The selection of stocks included in the portfolio followed criteria of liquidity and market relevance (JIA; ZHANG, 2024). The time series were processed and organized using Python libraries such as pandas and numpy, which are fundamental for data analysis in the Python language (WIĘCKOWSKI; KIZIELEWICZ; SAŁABUN, 2022). In addition, for the application of DEA, input variables (volatility and number of negative deviations) and output variables (return and number of positive deviations) were defined based on historical stock prices. Optimization was carried out using the SIAD software, considered effective for optimization applications in various studies (SILVA; SÁ, 2021; RIBEIRO; LONGARAY, 2022; SOUZA et al., 2022).

Different indicators were employed depending on the applied method. For the DEA model, performance variables considered statistical characteristics reflecting risk and return of assets, as recommended by Lartey, James, and Danso (2021). In the construction of the efficient portfolio, annualized average return, the covariance matrix, and asset standard deviation were used. For modeling repurchase as a financial option, inputs included the current stock price, investment amount, estimated volatility, time to maturity, and the risk-free rate. These indicators are fundamental for feeding the Black-Scholes-Merton and binomial models (RATIBENYAKOOL; NEAMMANEE, 2024).

The study employed a combination of robust quantitative techniques: (i) DEA for efficient asset selection; (ii) Monte Carlo simulations and portfolio optimization via the Markowitz model, considering the trade-off between risk and return; and (iii) financial option models (Black-Scholes and binomial tree) to measure the strategic value of repurchase. This combination allows the problem to be approached from different quantitative perspectives, thereby increasing the robustness of the results. The application of these techniques in finance is supported by authors such as Wang and Wang (2022), Ledoit and Wolf (2025), and Muroi and Suda (2022).

The research is quantitative in nature and follows the hypothetical-deductive method, starting from hypotheses in the corporate finance literature on share repurchase and efficient capital allocation (SEONG; NAM, 2022). It is an explanatory study with a computational approach (RODRIGUES et al., 2021). The case study is justified by the representativeness of the institution in the financial market and the frequency with which it carries out repurchases. Figure 1 presents the methodological framework of this study.

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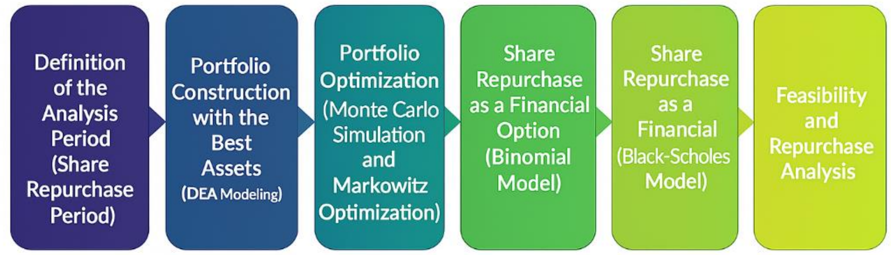

Figure 1. Methodological Framework.

The combination of the adopted techniques was guided by the nature of the problem, which involves corporate decisions under uncertainty. DEA was used to ensure that only efficiently performing assets were selected. The Markowitz portfolio provided an allocation alternative based on the maximization of risk-adjusted return. Such approaches are widely established in the literature (PETUKHINA et al., 2024). The next section presents the study results.

4. Results

This section aims to present and discuss the main results obtained from the comparative analysis between the share repurchase strategy by a bank and investment in an optimized stock portfolio. For this purpose, the data generated by the applied financial models are explored, including option valuation through the binomial and Black-Scholes models, as well as the construction of the efficient portfolio through the Markowitz model, with prior selection of the most efficient stocks according to Data Envelopment Analysis (output-oriented BCC DEA).

4.1. Data Envelopment Analysis for Asset Selection

For the selection of the ideal investment portfolio, the Data Envelopment Analysis (DEA) model was used with the objective of identifying the most efficient stocks. The sample consisted of the 10 stocks with the highest returns in 2023, based on B3 data. These stocks are presented in Table 1, and for the proper application of the DEA model, they were organized as Decision Making Units (DMUs), numbered from DMU 1 to DMU 10.

Table 1. Assets with the Highest Returns on the Brazilian Stock Exchange in 2023.

Decision-Making Unit	Ticker	Company Name	Economic Sector / Industry Segment
DMU1	YDUQ3.SA	Yduqs Participações S.A.	Education
DMU2	CMIN3.SA	CSN Mineração S.A.	Mining
DMU3	UGPA3.SA	Ultrapar Participações S.A.	Fuel Distribution / Logistics
DMU4	PETR4.SA	Petróleo Brasileiro S.A. (Petrobras)	Oil, Gas and Energy
DMU5	CYRE3.SA	Cyrela Brazil Realty S.A.	Civil Construction / Real Estate Development
DMU6	BBAS3.SA	Banco do Brasil S.A.	Financial Services / Banking
DMU7	PETR3.SA	Petróleo Brasileiro S.A. (Petrobras)	Oil, Gas and Energy
DMU8	IRBR3.SA	IRB Brasil Resseguros S.A.	Reinsurance / Insurance
DMU9	BRFS3.SA	BRF S.A.	Processed Foods / Agribusiness
DMU10	COGN3.SA	Cogna Educação S.A.	Education

Each asset was associated with a DMU (Decision-Making Unit) representing companies from different sectors of the economy, such as education, mining, energy, construction, financial services, and agribusiness. A correlation analysis (Figure 2) was conducted among the returns of the DMUs in order to evaluate the linear relationship between the assets. In general, weak and positive correlations were

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observed among most stocks, which is desirable for portfolio diversification. Notably, YDUQ3.SA and COGN3.SA, both from the education sector, showed the highest correlation between them (0.70).

Figure 2. Correlation Analysis among Assets.

For the application of the DEA model, input and output variables were defined to reflect aspects of risk and return of the analyzed assets. These variables are presented in Table 2, where the inputs are related to volatility and the number of negative standard deviations, while the outputs are associated with return and the number of positive standard deviations. The purpose of this selection is to evaluate the efficiency of the stocks considering the maximization of return.

Table 2. Performance Indicators of the Assets.

Variable	**Meaning**
Input 1	Volatility
Input 2	Number of Negative Standard Deviations
Output 1	Return
Output 2	Number of Positive Standard Deviations

The selected stocks exhibited distinct behaviors throughout 2023, as shown in Figure 3. It is noteworthy that YDUQ3.SA recorded the highest growth during the period, with sharp fluctuations, while COGN3.SA and IRBR3.SA showed the lowest values and lower volatility. It can also be observed that the assets display different behavior patterns, reflecting varying levels of risk and return.

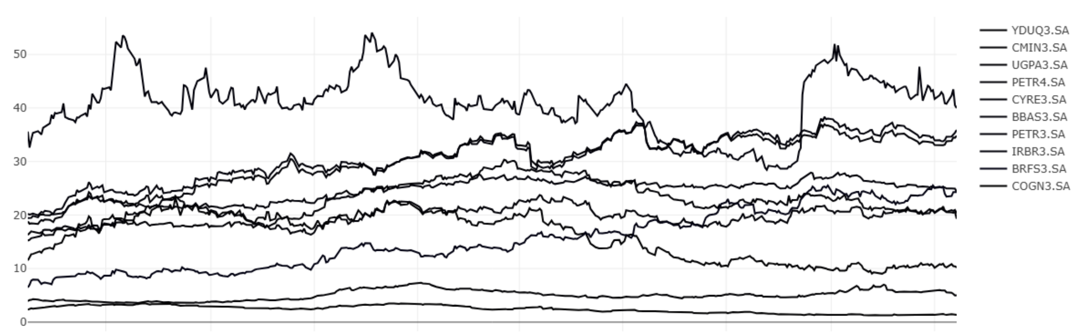

Figure 3. Price Behavior of the 10 Stocks with the Highest Returns in 2023.

To select the best stocks that would compose the portfolio, the BCC model was chosen, as it allows for variable returns to scale, which is more suitable for DMUs that behave in disproportionate ways (BANKER; CHARNES; COOPER, 1984). This approach is appropriate when considering stocks and economic sectors with significantly different prices within the sample. In this study, which was output-

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oriented, the inputs were kept constant while the outputs were maximized, as shown in Equations (1), (2), (3), and (4).

$$\max z = uy_i - u_i \quad (1)$$

$$s.a \ \sum_{m}^{t} x_i u_j = 1 \quad (2)$$

$$-vX + uY - u_0 e \leq 0 \quad (3)$$

$$u \geq 0, v \geq 0 \quad (4)$$

Em que z, e, e u_0 são escalares sem restrição de sinal. As matrizes u, v representam os pesos atribuídos às variáveis de saída e entrada, respectivamente. As matrizes Y e X contêm os dados de saída e entrada de todas as DMUs. As variáveis referem-se às entradas e saídas da DMU avaliada, ou seja, a i-ésima unidade. A Figura 3 mostra o resultado do modelo.

Where z, e, and u_0 are unrestricted scalars. The matrices uuu and vvv represent the weights assigned to the output and input variables, respectively. The matrices Y e X contain the output and input data of all DMUs. The variables refer to the inputs and outputs of the evaluated DMU, that is, the i-th unit. Figure 3 presents the result of the model.

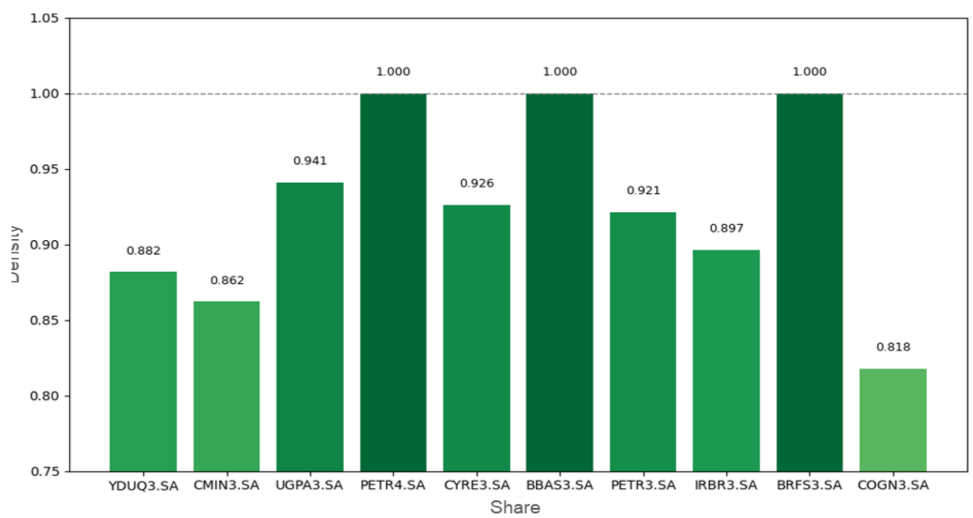

Figure 4. Efficiency of Stocks According to the Output-Oriented BCC Model.

Based on the DEA results presented in Figure 3, the five most efficient stocks are: PETR4.SA, BBAS3.SA, BRFS3.SA, UGPA3.SA, CYRE3.SA, and PETR3.SA.

4.2. Markowitz Model for Portfolio Optimization

Given the DEA results, these stocks will form the investment portfolio of this study. To optimize it, the Markowitz Efficient Portfolio model will be applied. First, a Monte Carlo Simulation was performed with 10,000 generated portfolios with distinct weights, as shown in the code presented in Figure 4.

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```
first_weights = []


for x in range(num_ports):
    # Weights
    weights = np.array(np.random.random(5))
    weights = weights / np.sum(weights)

    # Save first weight
    first_weights.append(weights[0])

    # Save weights
    all_weights[x, :] = weights

    # Expected return
    ret_arr[x] = np.sum((log_ret.mean() * weights))

    # Expected volatility
    vol_arr[x] = np.sqrt(np.dot(weights.T, np.dot(log_ret.cov(),
weights)))

    # Sharpe Ratio
    sharpe_arr[x] = ret_arr[x] / vol_arr[x]
```

Figure 5. Code for Monte Carlo Simulation for Random Portfolio Generation.

The generation of random samples aims to produce 10,000 Sharpe Ratios, which seek to balance risk and return, as shown in Equation 5.

$$SI = \frac{R_i}{\sigma_i} \quad (5)$$

Onde R_i e σ_i representam o Retorno e o Risco gerados, respectivamente. Essas variáveis são calculadas de acordo com as Equações 6 e 7.

Where R_i and σi\sigma_iσi represent the Return and the Risk generated, respectively. These variables are calculated according to Equations 6 and 7.

$$R_i = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (6)$$

$$\sigma_i = \sqrt{w^T \ \Sigma w} \quad (7)$$

Na Equação 6, foi utilizado o Log Retorno, mais adequado a análises econométricas para modelar os retornos dos preços P_i (SONG et al., 2022). Já na Equação 7, é calculada a volatilidade da carteira i, onde w^T é a matriz transposta dos pesos e Σw a matriz de variância covariância, onde os pesos w são gerados pela simulação de Monte Carlo de acordo com a Equação 8.

In Equation 6, the Log Return was used, which is more suitable for econometric analyses to model the returns of prices P_i (SONG et al., 2022). In Equation 7, the volatility of portfolio i is calculated,

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where w^T is the transposed weight matrix and Σw the variance-covariance matrix, with the weights

www generated through the Monte Carlo simulation according to Equation 8.

$$\sum_{i=1}^{n} w_i = 1 \tag{8}$$

The generation of random weights, as shown in Equation 8, must sum to 1. This means that, for each of the 10,000 portfolios, the sum of the 5 weights assigned to the selected assets must total 100%. Figure 6 shows the distribution of the weight generation, demonstrating that, on average, the weight distribution centers around 20% for each sample.

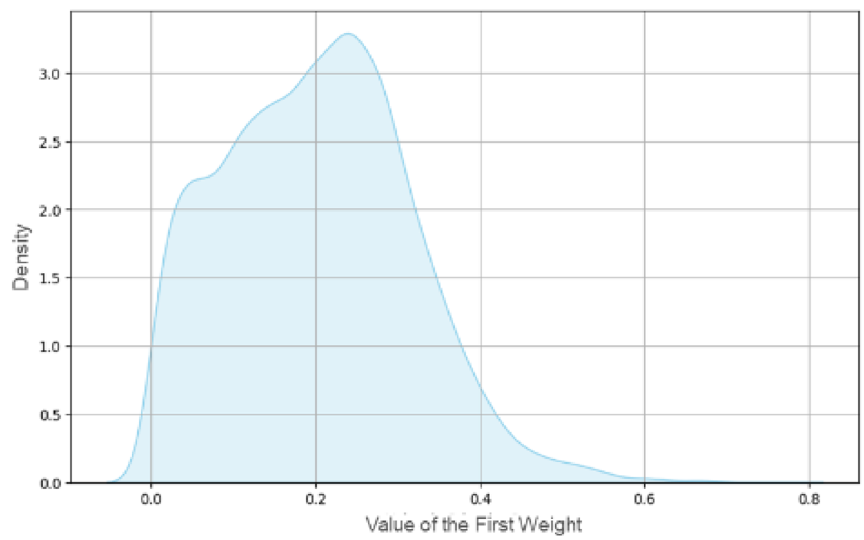

Figure 6. Behavior of Monte Carlo Simulation for Random Weight Generation.

After the Monte Carlo Simulation, with the randomly generated portfolios, the optimization of the efficient portfolio is then carried out through the Markowitz model. The optimization problem consists of minimizing the portfolio's risk, represented by the variance of the returns:

$$\min z = w^T \Sigma w \tag{9}$$

$s.a$

$$w^T 1 = 1 \tag{10}$$

$$w^T \mu = \mu_p \tag{11}$$

Onde μ é o vetor de retornos esperados, e μ_p representa o retorno desejado para a carteira. A primeira restrição garante que a soma dos pesos seja igual a 1, ou seja, 100% do capital está alocado, enquanto a segunda assegura que o retorno da carteira atinja um valor mínimo especificado. O Lagrangiano do problema é definido como mostra a Equação 12.

Where μ is the vector of expected returns, and μ_p represents the desired return for the portfolio. The first

constraint ensures that the sum of the weights equals 1, that is, 100% of the capital is allocated, while the

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second guarantees that the portfolio return reaches a specified minimum value. The Lagrangian of the problem is defined as shown in Equation 12.

$$L(\boldsymbol{w}, \boldsymbol{\lambda}, \boldsymbol{\gamma}) = w^T \boldsymbol{\Sigma w} - \boldsymbol{\lambda}(w^T \boldsymbol{1} - \boldsymbol{1}) - \boldsymbol{\gamma}\left(w^T \boldsymbol{\mu} - \boldsymbol{\mu_p}\right) \tag{12}$$

By differentiating with respect to www and setting it equal to zero, we obtain the first-order condition (F.O.C.), and the optimization is performed, as shown in Equation 13.

$$\boldsymbol{\Delta} L = \frac{\partial}{\partial w} w^T \boldsymbol{\Sigma w}^T - \frac{\partial}{\partial w} \boldsymbol{\lambda}(w^T \boldsymbol{1} - \boldsymbol{1}) - \frac{\partial}{\partial w} \boldsymbol{\gamma}\left(w^T \boldsymbol{\mu} - \boldsymbol{\mu_p}\right) = 0 \tag{13}$$

Multiplying both sides by the transposes 1^T and μ^T, we obtain a linear system to solve for the multipliers and optimize the portfolio, as shown in Equation 14.

$$\boldsymbol{\Sigma w} = \frac{\boldsymbol{\lambda}}{\boldsymbol{2}} + \frac{\boldsymbol{\gamma}}{\boldsymbol{2}} \mu \tag{14}$$

Figure 6 presents the result of the Markowitz model for this study. Aiming to find the balance between risk and return, the red point on the graph represents the optimal Sharpe Ratio, meaning that the efficient portfolio, with the optimal weights, lies at this point, overlaid on the efficient frontier in blue.

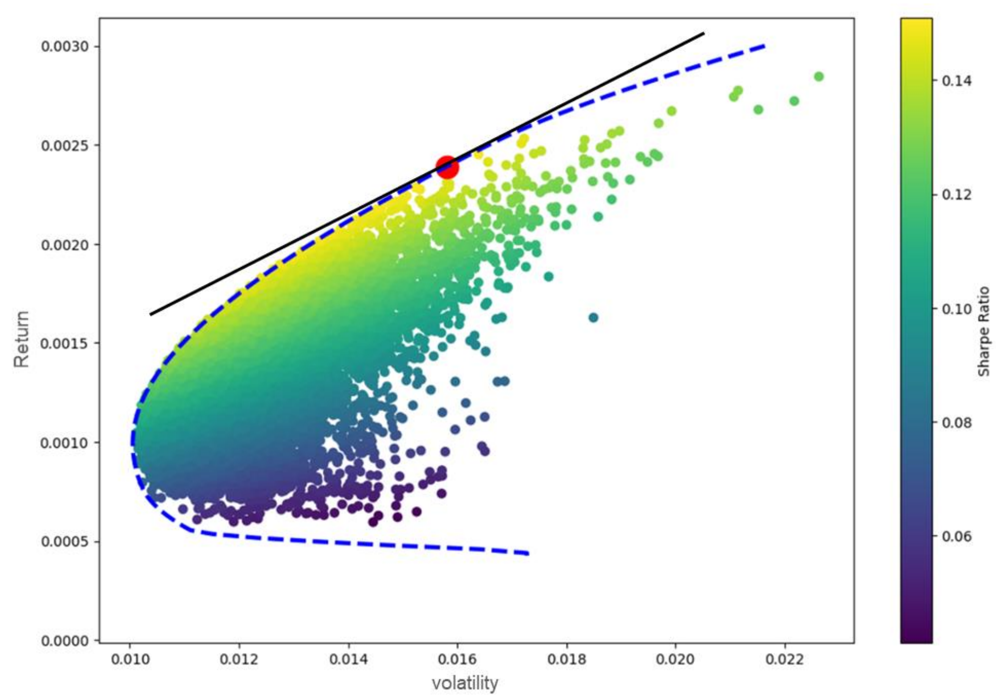

Figure 7. Asset Portfolio Optimization.

The optimal weights of the Markowitz efficient portfolio are presented in Table 3. In this table, it is possible to observe that, for a risk/return balance proposed by the Sharpe ratio, 51% of the investment should be allocated to PETR4.SA and 43% to BRFS3.SA. This result aligns with the DEA modeling performed, where the most efficient assets were these, along with BBAS3.SA, which receives the third-highest weight (9.6%), demonstrating that the two optimizations complement each other— the first selects the portfolio, and the second (Markowitz) determines the optimal weights.

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Table 3. Optimized Weights for the Efficient Portfolio.

Asset	Optimal Weight
PETR4.SA	0.514182
BBAS3.SA	0.009659
BRFS3.SA	0.434595
UGPA3.SA	0.000073
CYRE3.SA	0.041492

With the investment allocated to share repurchase directed toward this portfolio, the risk (standard deviation) is 4.71%, calculated according to Equation 7, and the expected return for the repurchase period is 23.32%, calculated according to Equation 15, where R is the portfolio return.

$$R = \sum_{I=1}^{n} w_i R_i \quad (15)$$

Modeled efficient portfolio during the repurchase period (379 days), a return of R$ 92,553,652.60 is obtained, totaling a profit of R$ 25,079,515.44. This is the value that will be compared to the results of the repurchase using Financial Options.

4.3. Financial Options for Share Repurchase Analysis

To analyze the profitability of the share repurchase, the value of dividends paid by the bank to shareholders during the analyzed period was considered. This is justified by the fact that, by repurchasing its own shares, the bank now holds part of its own capital, becoming indirectly the beneficiary of the dividends distributed. Therefore, the dividends paid represent an essential component in calculating the return of this strategy. The return from dividends can be observed in Figure 6.

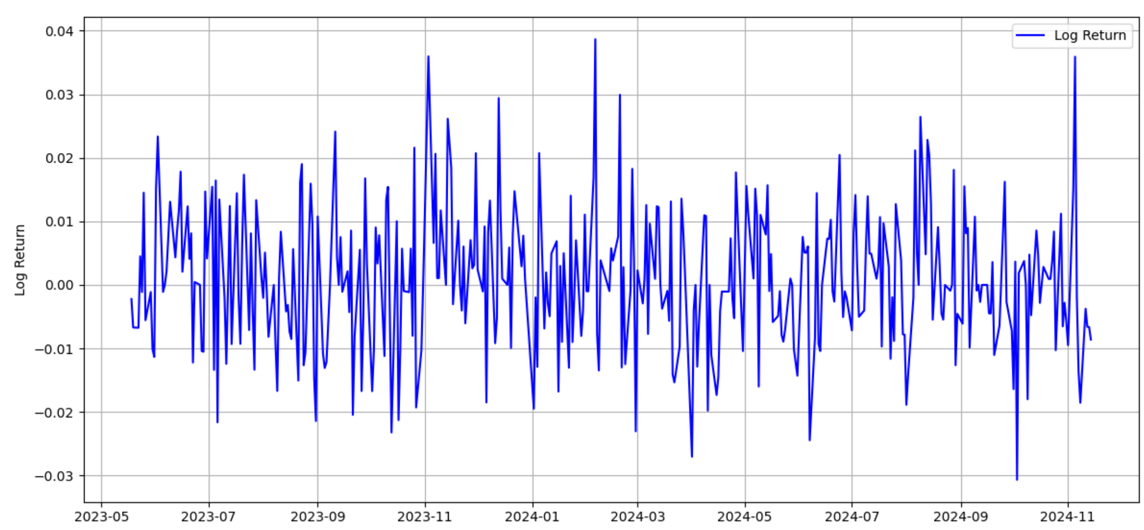

Figure 8. Log Returns of Dividends Over Time.

4.4. Binomial Model

To evaluate the investment in share repurchase, a model based on financial options was used. This approach treats the repurchase as a decision similar to exercising a call option, incorporating the uncertainty and time value of the asset. Among the possible models, the binomial model was initially chosen, which structures the evolution of the stock price over time through a recombination tree, allowing the repurchase value to be estimated under different market scenarios. Initially, the up and down factors for the stock price are defined based on volatility σ\sigmaσ and the time interval considered Δt (Equations 16 and 17).

$$U = e^{\sigma\sqrt{\Delta t}} \quad (16)$$

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$$d = \frac{1}{U} \tag{17}$$

Em seguida, calcula-se a probabilidade neutra ao risco, que representa a chance ajustada de o ativo subir, considerando a taxa livre de risco R_f, como mostra a equação 18.

Next, the risk-neutral probability is calculated, which represents the adjusted probability of the asset increasing, considering the risk-free rate R_f, as shown in Equation 18.

$$P = \frac{e^{R_f \Delta t} - d}{U - d} \tag{18}$$

Com esses parâmetros, constrói-se uma árvore binomial de preços, projetando os fluxos de caixa futuros esperados em cada nó da árvore. Por fim, o valor da opção no instante inicial ($t = 0$) é obtido pela comparação entre o valor presente dos fluxos futuros esperados e o custo de aquisição das ações, conforme a Equação 19, onde $\mathbb{R}^{\mathbb{Q}}$representa a expectativa sob a medida neutra ao risco, V_0 é o valor da ação no período de início da recompra vezes a quantidade e ações recompradas, e X o valor a ser pago na recompra. Essa abordagem permite mensurar se a recompra é vantajosa em relação a outras alternativas de alocação de capital.

With these parameters, a binomial price tree is constructed, projecting the expected future cash flows at each node of the tree. Finally, the option value at the initial time ($t = 0$) is obtained by comparing the present value of the expected future cash flows with the cost of acquiring the shares, as shown in Equation 19, where $\mathbb{R}^{\mathbb{Q}}$ represents the expectation under the risk-neutral measure, V_0 is the stock value at the start of the repurchase period multiplied by the number of shares repurchased, and X is the repurchase price. This approach allows for measuring whether the repurchase is advantageous compared to other capital allocation alternatives.

$$Option_{t=0} = \max\left\{\mathbb{R}^{\mathbb{Q}}\left[\frac{future\ cash\ flows}{\left(1 + R_f\right)^t}\right] V_0 - X\right\} \tag{19}$$

The analysis is divided into 3 steps, with $R_f = 14{,}75$, $\sigma = 0{,}92$ (volatility of dividends in the period), X = R$ 25,079,515.44 (the amount to abandon the repurchase and invest), V_0= R$ 67,474,137.16 (the product of the stock price at the time of repurchase and the number of shares repurchased, representing the invested amount). The value of abandoning the project X is the amount the investor would receive as a return by opting to invest in the Markowitz Optimal Portfolio. Figure 9 presents the binomial tree with the results of the model.

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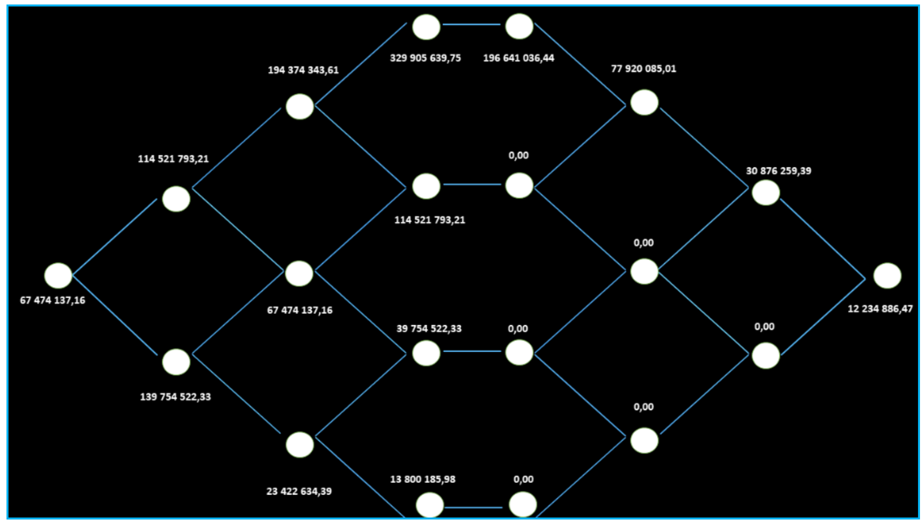

Figure 9. Binomial Tree of Financial Option for Share Repurchase Analysis.

The analysis of the binomial tree highlights the evolution of the potential values of the project over time, considering the uncertainty of prices. In the Project Price Tree (inverted), a wide range of final results is observed, with values ranging from R$ 13,800,185.98 to R$ 329,905,639.75. However, when applying the financial options methodology, the values from the Option Value Tree show that, despite the existence of positive scenarios at the top of the tree, most paths result in a zero value for the repurchase, reflecting insufficient future cash flows to justify exercising the option. The option value at the initial time was R$ 12,584,818.56, which is significantly lower than the project abandonment value—represented by the alternative investment in an efficient stock portfolio, which would generate R$ 25,079,515.44 in the same period. Thus, the analysis suggests that, from an economic-financial perspective, it would not be rational to opt for the share repurchase, as investing in the Markowitz portfolio would bring a higher return with the same capital.

4.5. Black & Scholes Models

To consolidate the viability analysis of the share repurchase, the Black & Scholes model was also applied. This model is widely used for pricing options in continuous time, considering variables such as volatility, time to maturity, risk-free rate, and the current value of the asset. Its application complements the results obtained from the binomial model, providing an additional estimate of the repurchase value as a financial option. The model works as follows: initially, it starts with the Black-Scholes partial differential equation, which describes the temporal evolution of the option value V as a function of the underlying asset price S, volatility σ\sigmaσ, and risk-free rate R_f.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0 \quad (20)$$

A solução analítica dessa equação para uma opção de compra europeia resulta na Equação 21, onde C é o valor da opção de compra, $S_0 = V_0$ o preço da recompra, k é o preço de exercício da opção, T o tempo de vencimento e $N(d_1)$ e $N(d_2)$ a função de distribuição acumulada da normal padrão.

The analytical solution to this equation for a European call option results in Equation 21, where C is the value of the call option, $S_0 = V_0$ is the repurchase price, k is the exercise price of the option, T is the time to maturity, and $N(d_1)$ and $N(d_2)$ are the cumulative distribution functions of the standard normal.

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad (21)$$

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The parameters d_1 and d_2 are given by Equations 22 and 23:

$$d_1 = \frac{\ln\frac{S_0}{k} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (22)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (23)$$

This formulation allows the calculation of the fair value of the repurchase as a financial option, considering the behavior of the market in continuous time. As a result, the call option value for the Black & Scholes model is R$ 20,118,768.17. This value is compared to the investment value in the **Markowitz** Efficient Portfolio. In conclusion, the most financially sound decision would be to invest in the stocks of the portfolio.

5. Discussion

This section aims to discuss the main results obtained throughout the study, in light of the proposed objective: comparing the financial attractiveness between the bank's share repurchase and the investment in an optimized portfolio. The application of the proposed methods allowed for a technical and well-founded demonstration that, in this specific case, investing in the Markowitz portfolio yields higher profitability compared to the share repurchase strategy.

5.1. Managerial Contributions

The managerial contributions of this work focus on the application of an integrated risk analysis approach, combining econometric, analytical, simulation, and optimization methods. This robust framework provides technical support for decision-making regarding share repurchase, allowing managers to quantitatively assess the financial impacts of this choice. Additionally, the proposed methodology can be replicated in different corporate contexts, strengthening the quality of the decision-making process.

5.2. Contributions to the Literature

The contributions to the literature of this work lie in the integrated application of techniques such as Data Envelopment Analysis (DEA), Markowitz portfolio optimization model, Monte Carlo simulation, and Financial Option pricing to share repurchase analysis. This methodological combination, still rarely explored together in this specific context, represents an innovative approach and significantly contributes to the advancement of the literature on corporate investment strategy evaluation, adding analytical rigor and versatility to financial analysis.

6. Conclusions

This study achieved its main objective by analyzing the economic viability of share repurchase by a bank, comparing this strategy with investment in an optimized stock portfolio. Through the integrated application of econometric models, simulation, optimization, and quantitative finance, it was possible to develop a robust technical analysis, evaluating the return and risk of each alternative. The analysis demonstrated that, in the studied scenario, investing in the efficient portfolio yielded a superior financial result compared to the repurchase, guiding decision-making based on quantitative evidence.

Several previous studies use tools such as portfolio analysis, option pricing models, and optimization techniques to evaluate financial strategies and support corporate decisions. Following this approach, this work incorporated established methods such as DEA-BCC, the Markowitz model, Monte Carlo simulation, and the binomial and Black & Scholes option models, innovatively integrating them into the analysis of share repurchases—an issue of high relevance in strategic financial management.

To verify the fulfillment of the proposed objective, real market data were collected and analyzed, with the mentioned models applied successively. Graphs and tables were used to represent the selection of stocks, the efficiency of the DMUs, the optimal portfolio composition, and the projected values through the binomial trees. The accumulated return of the optimized portfolio was compared to the value of the repurchase option,

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revealing that the alternative investment in stocks would surpass the repurchase value at the end of the analyzed period.

This study successfully demonstrated that, for the specific case analyzed, the share repurchase strategy would not be the most advantageous decision. The applied models showed that the expected value generated by the Markowitz portfolio exceeds the net present value of the repurchase, both in the binomial evaluation and through the Black & Scholes model. The approach used contributes to the literature by proposing a complete quantitative framework for strategic financial decisions, with potential application across different sectors.

The contributions of this research involve presenting an integrated methodology for evaluating corporate decisions involving share repurchase. Among the elements that reinforce its applicability are:

- Risk and return analysis through efficient asset selection with the DEA model;
- Construction of an optimal portfolio based on the Markowitz efficient frontier;
- Future scenario simulation with Monte Carlo for profitability projection;
- Pricing repurchases as real options, adding financial value to the decision.

These components provide managers with quantitative and replicable tools to support capital allocation decisions, comparing strategic alternatives in risk environments.

Despite its contributions, this study has some limitations. First, the analysis is centered on a single specific case, focusing on direct financial return, which limits the generalization of the results. Additionally, share repurchase is a strategic decision involving sensitive variables beyond immediate profitability—such as market signaling, capital structure, shareholder control, and long-term expectations. Therefore, a more robust analysis should consider, in addition to quantitative aspects, a qualitative evaluation with senior management, aligned with corporate objectives and strategic positioning. Future studies could expand this approach by incorporating interviews with decision-makers, dynamic market behavior models, and integration with ESG indicators, providing a more comprehensive and realistic view of the motivations and impacts of share repurchase.

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Appendix A. Python Code Used in the Analysis.

```
import pandas as pd
import streamlit as st
import plotly.express as px
from scipy.stats import norm
import seaborn as sns
import plotly.graph_objects as go
from PIL import Image
import subprocess
from yahooquery import Ticker
from datetime import datetime
import yfinance as yf

# Caminho para o seu arquivo
caminho = '/content/drive/MyDrive/Itaú/COTAHIST_A2023.TXT'

# Lista para armazenar os dados extraídos
dados = []

# Abrir e ler o arquivo
with open(caminho, 'r', encoding='latin1') as arquivo:
    for linha in arquivo:
        if linha.startswith('01'):  # apenas linhas com dados de ativos
            dados.append({
                'data': linha[2:10],
                'codigo_acao': linha[12:24].strip(),
                'nome_empresa': linha[27:39].strip(),
                'preco_abertura': int(linha[56:69]) / 100,
                'preco_maximo': int(linha[69:82]) / 100,
```

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```
        'preco_minimo': int(linha[82:95]) / 100,
        'preco_fechamento': int(linha[108:121]) / 100,
        'volume': int(linha[170:188])})
# Criar DataFrame
df = pd.DataFrame(dados)

# Converter a data para o formato datetime
df['data'] = pd.to_datetime(df['data'], format='%Y%m%d')

# Exibir as primeiras linhas
print(df.head())

df_hist = df
df_hist.head()
# Caminho para o arquivo (atenção aos caracteres especiais)
file_path = '/content/drive/MyDrive/Itaú/Preços ITAÚ.xlsx'

# Leitura da planilha
df = pd.read_excel(file_path, engine='openpyxl')

# Exibir as primeiras linhas do DataFrame
df.head()

df['LOG_RETURN'] = np.log(df['PRICE'] / df['PRICE'].shift(1))
df.head()
import matplotlib.pyplot as plt

# Converter a coluna de data (se ainda não estiver convertida)
df["DATE"] = pd.to_datetime(df["DATE"])

# Ordenar por data (caso os dados não estejam em ordem)
df = df.sort_values("DATE")

# Criar o gráfico
plt.figure(figsize=(12, 6))
plt.plot(df["DATE"], df["PRICE"], color='green', linewidth=2)

# Títulos e rótulos
plt.title("Preços no período de análise", fontsize=16)
plt.xlabel("Data de Fechamento", fontsize=12)
plt.ylabel("Preço (R$)", fontsize=12)

# Melhorar a visualização do eixo X
plt.xticks(rotation=45)

# Grade e exibição
plt.grid(True)
plt.tight_layout()
plt.show()

import numpy as np
from scipy.stats import lognorm
```

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```
# Garantir que 'PRICE' está limpo
precos = df['PRICE'].dropna()

# Ajustar a distribuição lognormal aos dados
shape, loc, scale = lognorm.fit(precos, floc=0)

# Criar intervalo de valores para a PDF ajustada
x = np.linspace(precos.min(), precos.max(), 1000)
pdf = lognorm.pdf(x, shape, loc, scale)

# Plotar
plt.figure(figsize=(10, 6))
plt.hist(precos, bins=30, density=True, alpha=0.6, color='skyblue', label='Dados reais')
plt.plot(x, pdf, 'r-', lw=2, label='Distribuição Lognormal ajustada')
plt.title('Distribuição Lognormal - Preço de Fechamento (Itaú)')
plt.xlabel('Preço de Fechamento')
plt.ylabel('Densidade de Probabilidade')
plt.legend()
plt.grid(True)
plt.show()
plt.figure(figsize=(12, 6))
plt.plot(df['DATE'], df['LOG_RETURN'], label='Log Return', color='blue')
plt.xlabel('Date')
plt.ylabel('Log Return')
plt.title('Log Returns ao Longo do Tempo')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()

# Garantir que a coluna DATE está no formato datetime
df['DATE'] = pd.to_datetime(df['DATE'])

# Criar a coluna 'MÊS' no formato 'YYYY-MM'
df['MÊS'] = df['DATE'].dt.to_period('M').astype(str)

# Agrupar por mês e calcular a **soma** dos log-retornos
df_mensal = df.groupby('MÊS')['LOG_RETURN'].sum().reset_index()

# Renomear a coluna de soma para 'RETORNO'
df_mensal.rename(columns={'LOG_RETURN': 'RETORNO'}, inplace=True)

# Filtrar os meses entre maio de 2023 e novembro de 2024
df_mensal = df_mensal[(df_mensal['MÊS'] >= '2023-05') & (df_mensal['MÊS'] <= '2024-11')]

df_mensal.head()
!pip install streamlit
!pip install yahooquery

acoes = ['VALE3.SA', 'MGLU3.SA', 'PETR4.SA', 'BBDC3.SA', 'WEGE3.SA', '^BVSP']
acoes
acoes = [
    'YDUQ3.SA',  # YDUQS PART ON NM
```

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```
    'CMIN3.SA',  # CSN MINERACAO ON N2
    'UGPA3.SA',  # ULTRAPAR ON NM
    'PETR4.SA',  # PETROBRAS PN
    'CYRE3.SA',  # CYRELA REALT ON NM
    'BBAS3.SA',  # BRASIL ON NM
    'PETR3.SA',  # PETROBRAS ON
    'IRBR3.SA',  # IRBBRASIL RE ON NM
    'BRFS3.SA',  # BRF SA ON NM
    'COGN3.SA'   # COGNA ON ON NM]

acoes
acoes_df = pd.DataFrame()
for acao in acoes:
    acoes_df[acao] = yf.download(acao,
            start='2023-05-16',  end='2024-11-16')['Close']
acoes_df.index = acoes_df.index.strftime('%Y-%m-%d')
acoes_df.reset_index(inplace=True)
acoes_df
acoes = acoes_df.copy()
#acoes.drop(labels = ['^BVSP'], axis=1, inplace=True)
figura = px.line(title = 'Histórico do preço das ações')
for i in acoes.columns[1:]:
  figura.add_scatter(x = acoes["Date"] ,y = acoes[i], name = i)
figura.show()
# Criar uma nova variável sem a coluna 'Date', se ela existir
if 'Date' in acoes_df.columns:
    acoes_sem_data = acoes_df.drop(columns=['Date'])
else:
    acoes_sem_data = acoes_df.copy()
# Tentar converter todos os valores para float, forçando erros a NaN
acoes_sem_data = acoes_sem_data.apply(pd.to_numeric, errors='coerce')
# Calculando o log-retorno diário
log_retornos = np.log(acoes_sem_data / acoes_sem_data.shift(1))

# Criando o DataFrame de resultados
indicadores = pd.DataFrame(index=acoes_sem_data.columns)

# OUTPUT: Retorno Anual (soma dos log-retornos diários)
indicadores['Retorno Anual (LogRetorno)'] = log_retornos.sum()

# OUTPUT: Quantidade de DP Positivo
indicadores['Quantidade de DP Positivo'] = (log_retornos > 0).sum()

# INPUT: Risco (Desvio Padrão dos log-retornos)
indicadores['Risco (Desvio Padrão DP)'] = log_retornos.std()

# INPUT: Quantidade de DP Negativo
indicadores['Quantidade de DP Negativo'] = (log_retornos < 0).sum()

# Exibir o resultado
print(indicadores)

# Dados conforme a imagem e mapeamento de DMUs para ações
```

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```
dados = {
    'Ação': [
        'YDUQ3.SA', 'CMIN3.SA', 'UGPA3.SA', 'PETR4.SA', 'CYRE3.SA',
        'BBAS3.SA', 'PETR3.SA', 'IRBR3.SA', 'BRFS3.SA', 'COGN3.SA'
    ], 'Eficiência': [ 0.881773, 0.862069, 0.940887, 1.000000, 0.926108,
        1.000000, 0.921355, 0.896552, 1.000000, 0.817734]}

# Criar o DataFrame
acoes_dea = pd.DataFrame(dados)

# Função para converter a eficiência em cor (vermelho a verde)
def cor_verde_vermelho(valor):
    # Interpola entre vermelho (baixa eficiência) e verde (alta)
    return mcolors.to_hex(plt.cm.RdYlGn(valor))

# Criar as cores para as barras
cores = [cor_verde_vermelho(valor) for valor in acoes_dea['Eficiência']]

# Plotar o gráfico
plt.figure(figsize=(10, 6))
barras = plt.bar(acoes_dea['Ação'], acoes_dea['Eficiência'], color=cores)
plt.ylim(0.75, 1.05)
plt.axhline(y=1.0, color='gray', linestyle='--', linewidth=1)
plt.title('Eficiência das Ações - DEA BCC Orientado a Output')
plt.ylabel('Eficiência')
plt.xlabel('Ação')

# Adiciona os valores acima das barras
for barra, valor in zip(barras, acoes_dea['Eficiência']):
    plt.text(barra.get_x() + barra.get_width()/2, barra.get_height() + 0.01,
             f"{valor:.3f}", ha='center', va='bottom', fontsize=9)

plt.tight_layout()
plt.show()

# Passo 1: Obter os nomes das 5 ações mais eficientes a partir do DataFrame acoes_dea
top5_acoes = acoes_dea.nlargest(5, 'Eficiência')['Ação'].tolist()

# Passo 2: Selecionar essas colunas no acoes_df (incluindo a coluna 'Date')
colunas_selecionadas = ['Date'] + top5_acoes

# Passo 3: Criar a nova variável com os dados filtrados
acoes_eficientes_dea = acoes_df[colunas_selecionadas].copy()
acoes_eficientes_dea.head()
dataset = acoes_df.copy()
dataset.drop(labels = ['Date'], axis=1, inplace=True)
taxas_retorno = np.log(dataset / dataset.shift(1))
dataset_date = acoes_df.copy()
date = dataset_date.filter(["Date"])
taxas_retorno = pd.concat([date, taxas_retorno], axis=1)
taxas_retorno
taxas_retorno.describe()
taxas_retorno.select_dtypes(include=['number']).mean() * 100
```

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```
figura = px.line(title = 'Histórico de retorno do Portfólio')
for i in taxas_retorno.columns[1:]:
  figura.add_scatter(x = taxas_retorno["Date"] ,y = taxas_retorno[i], name = i)
figura.show()
correlacao = taxas_retorno.select_dtypes(include=['number']).corr()
# Arredondando os valores para 2 casas decimais
correlacao = np.round(correlacao, 2)
correlacao
# Definindo o esquema de cores com vermelho em valores altos (próximo a 1)
custom_colorscale = [
    [0.0, 'green'],   # Cor para o limite inferior (-1)
    [0.5, 'blue'],  # Cor para o valor neutro (0)
    [1.0, 'red']     # Cor para o limite superior (1)
]

# Criando o heatmap com o esquema de cores customizado
fig = px.imshow(correlacao,
                text_auto=True,
                aspect="auto",
                color_continuous_scale=custom_colorscale,  # Esquema de cores customizado
                labels=dict(color="Correlações"),
                zmin=-1, zmax=1)  # Definindo o limite de correlações

# Exibindo o gráfico
fig.show()
acoes_port = acoes_eficientes_dea.copy()
#acoes_port.drop(labels = ['^BVSP'], axis=1, inplace=True)
log_ret = acoes_port.copy()
log_ret.drop(labels = ["Date"], axis = 1, inplace = True)
log_ret = np.log(log_ret/log_ret.shift(1))
np.random.seed(42)
num_ports = 50000
all_weights = np.zeros((num_ports, len(acoes_port.columns[1:])))
ret_arr = np.zeros(num_ports)
vol_arr = np.zeros(num_ports)
sharpe_arr = np.zeros(num_ports)
# Número de portfólios simulados
num_ports = 10000

# Inicialização dos arrays
all_weights = np.zeros((num_ports, 5))
ret_arr = np.zeros(num_ports)
vol_arr = np.zeros(num_ports)
sharpe_arr = np.zeros(num_ports)

# Lista para armazenar o primeiro peso de cada simulação
first_weights = []

for x in range(num_ports):
    # Weights
    weights = np.array(np.random.random(5))
    weights = weights / np.sum(weights)
```

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```
    # Save first weight
    first_weights.append(weights[0])

    # Save weights
    all_weights[x, :] = weights

    # Expected return
    ret_arr[x] = np.sum((log_ret.mean() * weights))

    # Expected volatility
    vol_arr[x] = np.sqrt(np.dot(weights.T, np.dot(log_ret.cov(), weights)))

    # Sharpe Ratio
    sharpe_arr[x] = ret_arr[x] / vol_arr[x]

# Plotando o gráfico de densidade do primeiro peso
plt.figure(figsize=(10,6))
sns.kdeplot(first_weights, fill=True, color='skyblue')
plt.title('Densidade de Probabilidade do Primeiro Peso')
plt.xlabel('Valor do Primeiro Peso')
plt.ylabel('Densidade')
plt.grid(True)
plt.show()
print("Max Sharpe Ratio: {}". format(sharpe_arr.max()))
print("Local do Max Sharpe Ratio: {}". format(sharpe_arr.argmax()))
# Pesos do Portfólio do Max Sharpe Ratio
melhores_pesos = all_weights[sharpe_arr.argmax(),:]
print(melhores_pesos)
# salvando os dados do Max Sharpe Ratio
max_sr_ret = ret_arr[sharpe_arr.argmax()]
max_sr_vol = vol_arr[sharpe_arr.argmax()]
print(max_sr_ret*100)
print(max_sr_vol*100)
plt.figure(figsize=(12,8))
plt.scatter(vol_arr, ret_arr, c=sharpe_arr, cmap='viridis')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatilidade')
plt.ylabel('Retorno')
plt.scatter(max_sr_vol, max_sr_ret,c='red', s=200) # black dot
plt.show()
def get_ret_vol_sr(weights):
    weights = np.array(weights)
    ret = np.sum(log_ret.mean() * weights)
    vol = np.sqrt(np.dot(weights.T, np.dot(log_ret.cov(), weights)))
    sr = ret/vol
    return np.array([ret, vol, sr])

def neg_sharpe(weights):
# the number 2 is the sharpe ratio index from the get_ret_vol_sr
    return get_ret_vol_sr(weights)[2] * -1

def check_sum(weights):
    #return 0 if sum of the weights is 1
```

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```
    return np.sum(weights)-1
cons = ({'type': 'eq', 'fun': check_sum})
bounds = ((0,1), (0,1), (0,1), (0,1), (0,1))
init_guess = ((0.2),(0.2),(0.2),(0.2),(0.2))
frontier_y = np.linspace(0.0030, 0.00013, 150)
def minimize_volatility(weights):
    return get_ret_vol_sr(weights)[1]
from scipy import optimize
frontier_x = []

for possible_return in frontier_y:
    cons = ({'type':'eq', 'fun':check_sum},
            {'type':'eq', 'fun': lambda w: get_ret_vol_sr(w)[0] - possible_return})

    result = optimize.minimize(minimize_volatility,init_guess,method='SLSQP', bounds=bounds, constraints=cons)
    frontier_x.append(result['fun'])
plt.figure(figsize=(12,8))
plt.scatter(vol_arr, ret_arr, c=sharpe_arr, cmap='viridis')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatilidade')
plt.ylabel('Return')
plt.plot(frontier_x,frontier_y, 'b--', linewidth=3)
plt.scatter(max_sr_vol, max_sr_ret,c='red', s=250)
# plt.savefig('cover.png')
plt.show()
acoes_port = acoes_eficientes_dea.copy()
#acoes_port.drop(labels = ['^BVSP'], axis=1, inplace=True)
acoes_port
def alocacao_ativos(dataset, dinheiro_total, seed=0, melhores_pesos=[]):
    dataset = dataset.copy()

    if seed != 0:
        np.random.seed(seed)

    if len(melhores_pesos) > 0:
        pesos = melhores_pesos
    else:
        pesos = np.random.random(len(dataset.columns) - 1)
        pesos = pesos / pesos.sum()

    colunas = dataset.columns[1:]

    for i in colunas:
        dataset[i] = (dataset[i] / dataset[i][0])

    for i, acao in enumerate(dataset.columns[1:]):
        dataset[acao] = dataset[acao] * pesos[i] * dinheiro_total

    dataset['soma valor'] = dataset.select_dtypes(include=['number']).sum(axis=1)

    datas = dataset['Date']
    dataset.drop(labels=['Date'], axis=1, inplace=True)
```

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```
    dataset['taxa retorno'] = 0.0

    for i in range(1, len(dataset)):
        dataset.loc[i, 'taxa retorno'] = np.log(dataset.loc[i, 'soma valor'] / dataset.loc[i - 1, 'soma valor']) * 100

    acoes_pesos = pd.DataFrame({'Ações': colunas, 'Pesos': pesos})

    return dataset, datas, acoes_pesos, dataset.loc[len(dataset) - 1, 'soma valor']
acoes_pesos
# Remove a coluna de data
retornos = acoes_port.iloc[:, 1:]

# Alinha os pesos com as colunas dos retornos, usando merge com base no nome das ações
# (isso garante que os pesos estão na mesma ordem dos retornos)
pesos_ordenados = (
    pd.merge(pd.DataFrame(retornos.columns, columns=['Ações']),
             acoes_pesos,
             on='Ações',
             how='left')
    ['Pesos']
    .values
    .astype(float)
)

# Cálculo do retorno médio da carteira (média dos retornos de cada ação ponderada pelos pesos)
retorno_medio_carteira = np.dot(retornos.mean().values, pesos_ordenados)

# Matriz de covariância dos retornos
matriz_cov = retornos.cov().values

# Cálculo do risco da carteira (desvio padrão)
risco_carteira = np.sqrt(np.dot(pesos_ordenados.T, np.dot(matriz_cov, pesos_ordenados)))

# Exibe os resultados
retorno_medio_carteira, risco_carteira

# Retorno anual (exemplo: 0.2332 para 23,32%)
retorno_anual = retorno_medio_carteira/100

# Converter para retorno diário equivalente
retorno_diario = (1 + retorno_anual) ** (1/252) - 1
print(retorno_diario)

# Valor inicial investido
valor_inicial = 67474137.16

# Número de períodos (dias úteis)
T = len(acoes_port)

# Calcular valor final com capitalização composta
valor_final_markowitz = valor_inicial * (1 + retorno_diario) ** T

# Exibir resultado
```

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```
valor_final_markowitz

soma_valor
figura = px.line(x = datas, y = dataset['taxa retorno'], title = 'Retorno diário do portfólio',
              labels=dict(x="Data", y="Retorno %"))
figura.add_hline(y = dataset['taxa retorno'].mean(), line_color="red", line_dash="dot", )
figura.show()
import plotly.express as px

fig_box = px.box(
    dataset,
    y='taxa retorno',
    title='Distribuição dos Retornos (Boxplot)',
    labels={'taxa retorno': 'Retorno %'}
)

fig_box.show()

# 1. Obter os dados de retorno (série real)
dados_retorno = dataset['taxa retorno'].dropna()

# 2. Ajustar a distribuição de Cauchy aos dados
params = cauchy.fit(dados_retorno)  # Retorna (loc, scale)
loc, scale = params

# 3. Gerar valores de x para o gráfico
x = np.linspace(dados_retorno.min(), dados_retorno.max(), 1000)
pdf_cauchy = cauchy.pdf(x, loc=loc, scale=scale)

# 4. Plotar a comparação
plt.figure(figsize=(10, 6))

# Distribuição empírica (histograma com densidade)
sns.histplot(dados_retorno, kde=True, stat='density', bins=50, label='Distribuição Empírica', color='skyblue', edgecolor='black')

# Distribuição de Cauchy ajustada
plt.plot(x, pdf_cauchy, label='Distribuição de Cauchy Ajustada', color='darkred', linewidth=2)

# Configurações do gráfico
plt.title('Ajuste da Distribuição de Cauchy aos Retornos')
plt.xlabel('Retorno (%)')
plt.ylabel('Densidade de Probabilidade')
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()

figura = px.line(title = 'Evolução do patrimônio')
for i in dataset.drop(columns = ['soma valor', 'taxa retorno']).columns:
  figura.add_scatter(x = datas, y = dataset[i], name = i)
figura.show()
figura = px.line(x = datas, y = dataset['soma valor'],
```

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```
                title = 'Evolução do patrimônio da Carteira',
                labels=dict(x="Data", y="Valor R$"))
figura.add_hline(y = dataset['soma valor'].mean(),
                line_color="green", line_dash="dot", )
figura.add_hline(y = 10000,
                line_color="red", line_dash="dot", )
figura.show()
D = len(df)
Q = 10000000 #Quandidades de ações recompradas (16/05/2023 - 16/11/2024)
p0 = df['PRICE'].iloc[0] #Preço da ação no primeiro dia da recompra (16/05/2023)
V0 = Q*p0 #Valor investido no dia (16/05/2023)
V0
retorno_total = np.log(dataset['soma valor'].iloc[-1] / dataset['soma valor'].iloc[0]) * 100

valor_final = V0 * (1 + retorno_total / 100)
lucro_total = valor_final - V0
retorno_percentual = (valor_final / V0 - 1) * 100

print(f"Valor final da carteira: R$ {valor_final:,.2f}")
print(f"Lucro total: R$ {lucro_total:,.2f}")
print(f"Retorno percentual: {retorno_percentual:.2f}%")
Ri = max_sr_ret*100 #Retorno da carteira ótima
#Rm = dados['Rm'].sum() #Retorno de Mercado
Rf = 0.1375 #Taxa livre de risco (26/05/2025)
sigma_carteira = max_sr_vol*100 #Volatilidade da carteira ótima
print(f"Retorno da carteira ótima (Ri): {Ri:.2f}")
#print(f"Retorno de Mercado (Rm): {Rm:.2f}")
print(f"Taxa livre de risco (Rf): {Rf:.2f}")
print(f"Volatilidade da carteira ótima (sigma): {sigma_carteira:.2f}")
V0 = Q*p0 #Valor presente (16/05/2023)
VF = V0 * (1 + max_sr_ret*100)**1  # como é 1 ano
print(f"Valor futuro após 1 ano: R$ {VF:.2f}")

# Converter retorno anual em retorno mensal composto
retorno_mensal = (1 + max_sr_ret*100) ** (1 / 12) - 1

# Valor futuro após 18 meses de capitalização composta
VF = V0 * (1 + retorno_mensal) ** 18

# Imprimir resultados
def formatar_real(valor):
   return f"R$ {valor:,.2f}".replace(",", "X").replace(".", ",").replace("X", ".")

print("Retorno mensal equivalente:", f"{retorno_mensal*100:.2f}%")
print("Valor Futuro após 18 meses:", formatar_real(VF))

#Ações ITAUSA entre 15/05/2023 e 16/11/2024
p = df['PRICE'].mean() #Preço médio (16/05/2023 - 16/11/2024)
sigma = df['PRICE'].std() #Desvio padrão (16/05/2023 - 16/11/2024)

x = VF # Valor da Recompra
p0 = df['PRICE'].iloc[0] #Preço da ação no primeiro dia da recompra (16/05/2023)
V0 = Q*p0 #Valor presente (16/05/2023)
```

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```
Rf = 0.1475 #Taxa livre de risco (26/05/2025)

# Exibir os resultados
print(f"Preço médio (p): {p:.2f}")
print(f"Volatilidade (sigma): {sigma:.2f}")
print(f"Valor total da recompra (x): R$ {x:,.2f}")
print(f"Preço da ação ao iniciar a recompra (p0): R$ {p0:.2f}")
print(f"Valor presente da recompra (V0): R$ {V0:,.2f}")

# Soma dos Retornos da Itaúsa
Ri = df['LOG_RETURN'].sum()
print(Ri)
# Cálculos
lucro = V0 * Ri
montante_final = V0 + lucro
percentual_ganho = V0/lucro

# Resultados
print(f"Lucro gerado: R$ {lucro:.2f}")
print(f"Percentual de ganho: {percentual_ganho:.2f}%")
print(f"Montante final: R$ {montante_final:.2f}")

 #Ações ITAUSA entre 15/05/2023 e 16/11/2024
p = df['PRICE'].mean() #Preço médio (16/05/2023 - 16/11/2024)
sigma = df['PRICE'].std() #Desvio padrão (16/05/2023 - 16/11/2024)
Q = 10000000 #Quandidades de ações recompradas (16/05/2023 - 16/11/2024)
x = valor_final_markowitz # Valor de abandono da recompra e investimento no portfólio ótimo
p0 = df['PRICE'].iloc[0] #Preço da ação no primeiro dia da recompra (16/05/2023)
V0 = Q*p0 #Valor presente (16/05/2023)
Rf = 0.1475 #Taxa livre de risco (26/05/2025)

# Exibir os resultados
print(f"Preço médio (p): {p:.2f}")
print(f"Volatilidade (sigma): {sigma:.2f}")
print(f"Valor total da recompra (x): R$ {x:,.2f}")
print(f"Preço da ação ao iniciar a recompra (p0): R$ {p0:.2f}")
print(f"Valor presente da recompra (V0): R$ {V0:,.2f}")


T = 1     # Tempo até o vencimento da opção (em anos)
N = 3     # Número de passos na árvore binomial

# Parâmetros derivados
dt = T / N
u = np.exp(sigma * np.sqrt(dt))  # Fator de subida
d = 1 / u                        # Fator de descida
p = (np.exp(Rf * dt) - d) / (u - d)  # Probabilidade neutra ao risco

# Construção da árvore de preços
project_values = np.zeros((N+1, N+1))

for j in range(N+1):
    for i in range(j+1):
```

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```
        project_values[i, j] = V0 * (u ** (j - i)) * (d ** i)

# Árvore de opção (valor da opção em cada nó)
option_values = np.zeros_like(project_values)

# Valor da opção no vencimento: max(V - K, 0)
for i in range(N+1):
    option_values[i, N] = max(project_values[i, N] - x, 0)

# Retorno da opção pelos passos anteriores
for j in range(N-1, -1, -1):
    for i in range(j+1):
        hold = np.exp(-Rf * dt) * (p * option_values[i, j+1] + (1 - p) * option_values[i+1, j+1])
        exercise = max(project_values[i, j] - x, 0)
        option_values[i, j] = max(hold, exercise)

# Valor da opção hoje
print(f"Valor da opção real de adiar o projeto: R$ {option_values[0, 0]:.2f}")

def print_inverted_tree(tree, title="Árvore", node_width=12, v_spacing=1):
    N = tree.shape[1] - 1
    print(f"{title} (invertida):\n")
    for j in range(N, -1, -1):  # Começa do último tempo (inverso)
        padding = " " * ((N - j) * node_width // 2)
        print(padding, end="")
        for i in range(j + 1):
            print(f"{tree[i, j]:>{node_width}.2f}", end=" " * node_width)
        print()
        for _ in range(v_spacing):
            print()

# Chamadas
print_inverted_tree(project_values, title="Árvore de Preços do Projeto", v_spacing=1)
print()  # Apenas um espaço entre as árvores
print()  # Apenas um espaço entre as árvores
print_inverted_tree(option_values, title="Árvore de Valores da Opção", v_spacing=1)


def plot_binomial_tree(tree, title="Árvore", node_color="skyblue", text_color="black"):
    N = tree.shape[1] - 1
    fig, ax = plt.subplots(figsize=(12, 6))
    ax.set_title(title, fontsize=14)
    ax.axis("off")

    node_positions = {}  # Para armazenar posições dos nós

    # Plot dos nós e textos
    for j in range(N + 1):  # Tempo
        for i in range(j + 1):  # Nível (descidas)
            x = j
            y = -i  # negativo para que a árvore "cresça para baixo"
            node_positions[(i, j)] = (x, y)
            ax.plot(x, y, 'o', color=node_color, markersize=12)
```

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```
        ax.text(x, y, f"{tree[i, j]:.2f}", ha='center', va='center', fontsize=8, color=text_color)

    # Conexões entre nós (arestas da árvore)
    for j in range(N):
        for i in range(j + 1):
            x0, y0 = node_positions[(i, j)]
            x1, y1 = node_positions[(i, j + 1)]
            x2, y2 = node_positions[(i + 1, j + 1)]

            ax.plot([x0, x1], [y0, y1], color='gray', linewidth=1)
            ax.plot([x0, x2], [y0, y2], color='gray', linewidth=1)

    plt.tight_layout()
    plt.show()
plot_binomial_tree(project_values, title="Árvore de Preços do Projeto", node_color="skyblue")
plot_binomial_tree(option_values, title="Árvore de Valores da Opção", node_color="lightgreen")


from scipy.stats import norm

# Parâmetros do modelo
S0 = V0   # Valor atual do projeto
K = VF    # Custo do investimento (preço de exercício)           # Tempo até o vencimento (em anos)
r = Rf        # Taxa livre de risco

# Cálculo dos parâmetros d1 e d2
d1 = (np.log(S0 / K) + (r + 0.5 * sigma ** 2) * T) / (sigma * np.sqrt(T))
d2 = d1 - sigma * np.sqrt(T)

# Cálculo do valor da opção (call)
call_price = S0 * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2)

# Exibir o resultado
print(f"Valor da opção real de adiar o projeto (Black-Scholes): R$ {call_price:,.2f}")

# Substitua com seus valores reais
valor_binomial = option_values[0, 0]
call_price = 22500000.00  # ou substitua pela variável real calculada

# Criar DataFrame
tabela = pd.DataFrame({
    "Modelo": ["Modelo Binomial", "Modelo Black & Scholes"],
    "Valor de Adiar a Recompra até o fim": [valor_binomial, call_price]
})

# Formatar com separador de milhar e duas casas decimais
tabela["Valor de Adiar a Recompra até o fim"] = tabela["Valor de Adiar a Recompra até o fim"].apply(
    lambda x: f"R$ {x:,.2f}".replace(",", "X").replace(".", ",").replace("X", ".")
)

# Exibir a tabela
print(tabela.to_string(index=False))
```

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```
# Seus valores
valor_binomial = option_values[0, 0]
call_price = 22500000.00  # Substitua com a variável real, se preferir

# Criar DataFrame
df = pd.DataFrame({
    "Modelo": ["Modelo Binomial", "Modelo Black & Scholes"],
    "Valor de Adiar a Recompra até o fim": [valor_binomial, call_price]
})

# Formatando os valores
df["Valor de Adiar a Recompra até o fim"] = df["Valor de Adiar a Recompra até o fim"].apply(
    lambda x: f"R$ {x:,.2f}".replace(",", "X").replace(".", ",").replace("X", ".")
)

# Criar figura
fig, ax = plt.subplots(figsize=(8, 2))  # Tamanho ajustável
ax.axis('off')  # Esconde os eixos

# Tabela com estilo
table = ax.table(
    cellText=df.values,
    colLabels=df.columns,
    loc='center',
    cellLoc='center',
    colColours=["#40466e", "#40466e"],
    colWidths=[0.4, 0.6]
)

# Estilo da tabela
table.auto_set_font_size(False)
table.set_fontsize(12)
table.scale(1.2, 1.5)

# Cor das células
for i in range(len(df)):
    for j in range(len(df.columns)):
        cell = table[(i+1, j)]
        cell.set_facecolor('#f2f2f2' if i % 2 == 0 else '#ffffff')

# Título
plt.title("Valor da Opção de Adiar a Recompra", fontsize=14, fontweight='bold')

# Exibir
plt.tight_layout()
plt.show()
jdjopdsjfospdf
```

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